On the First Non-zero Stekloff Eigenvalues

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Abstract

Let $(M, \langle , \rangle)$ be an $n(\geq 2)$-dimensional compact Riemannian manifold with boundary and non-negative Ricci curvature. Consider the following two Stekloff eigenvalue problems

\[
\Delta u = 0 \text{ in } M, \quad \frac{\partial u}{\partial \nu} = pu \quad \text{on } \partial M;
\]
\[
\Delta^2 u = 0 \text{ in } M, \quad u = \Delta u - q\frac{\partial u}{\partial \nu} = 0 \quad \text{on } \partial M;
\]

where $\Delta$ is the Laplacian operator on $M$ and $\nu$ denotes the outward unit normal on $\partial M$. The first non-zero eigenvalues of the above problems will be denoted by $p_1$ and $q_1$, respectively. We prove that if the principle curvatures of the second fundamental form of $\partial M$ are bounded below by a positive constant $c$, then

\[
p_1 \leq \frac{\sqrt{\lambda_1}}{(n-1)c} \left( \sqrt{\lambda_1} + \sqrt{\lambda_1 - (n-1)c^2} \right)
\]

with equality holding if and only if $\Omega$ is isometric to an $n$-dimensional Euclidean ball of radius $\frac{1}{c}$, here $\lambda_1$ denotes the first non-zero eigenvalue of the Laplacian of $\partial M$. We also show that if the mean curvature of $\partial M$ is bounded below by a positive constant $c$ then $q_1 \geq nc$ with equality holding if and only if $M$ is isometric to an $n$-dimensional Euclidean ball of radius $\frac{1}{c}$ (joint work with Changyu Xia).