On the First Non-zero Stekloff Eigenvalues

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Abstract

Let $(M, \langle , \rangle)$ be an $n(\geq 2)$-dimensional compact Riemannian manifold with boundary and non-negative Ricci curvature. Consider the following two Stekloff eigenvalue problems

$$\Delta u = 0 \ \text{ in } \ M, \quad \frac{\partial u}{\partial \nu} = pu \qquad \text{on } \ \partial M;$$
$$\Delta^2 u = 0 \ \text{ in } \ M, \quad u = \Delta u - q\frac{\partial u}{\partial \nu} = 0 \qquad \text{on } \ \partial M;$$

where Δ is the Laplacian operator on M and ν denotes the outward unit normal on ∂M. The first non-zero eigenvalues of the above problems will be denoted by p_1 and q_1, respectively. We prove that if the principle curvatures of the second fundamental form of ∂M are bounded below by a positive constant c, then

$$p_1 \leq \frac{\sqrt{\lambda_1}}{(n-1)c}\left(\sqrt{\lambda_1} + \sqrt{\lambda_1 - (n-1)c^2}\right)$$

with equality holding if and only if Ω is isometric to an n-dimensional Euclidean ball of radius $\frac{1}{c}$, here λ_1 denotes the first non-zero eigenvalue of the Laplacian of ∂M. We also show that if the mean curvature of ∂M is bounded below by a positive constant c then $q_1 \geq nc$ with equality holding if and only if M is isometric to an n-dimensional Euclidean ball of radius $\frac{1}{c}$ (joint work with Changyu Xia).