Recent developments in universal estimates for eigenvalues of a biharmonic operator

Qing-Ming Cheng

Saga University

In this talk, we deal with two eigenvalue problems of a biharmonic operator, which are called a buckling problem and a clamped plate problem, respectively.

Let M be an n-dimensional complete Riemannian manifold, Ω a bounded domain with piecewise smooth boundary $\partial\Omega$ in M. The following eigenvalue problem is called a buckling problem:

$$\begin{cases} \Delta^2 u = -\Lambda \Delta u \quad in \quad \Omega, \\ u|_{\partial\Omega} = \left.\dfrac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0, \end{cases}$$

where Δ is the Laplacian on M and ν denotes the unit outward normal to the boundary $\partial\Omega$ of Ω.

In 1955, Payne, Pólya and Weinberger proposed the following open problem on universal inequalities for eigenvalues of the buckling problem, when M is a Euclidean space:

The problem of PPW. Whether can one obtain a universal inequality for eigenvalues of the buckling problem, which is similar to the universal inequality for eigenvalues of the Dirichlet eigenvalue problem of the Laplacian?

We will consider this problem and give its solution according to my joint work with professor Yang.

Secondly, we study the Dirichlet eigenvalue problem of the biharmonic operator, which is also called a clamped plate problem:

$$\begin{cases} \Delta^2 u = \Gamma u \quad in \quad \Omega, \\ u|_{\partial\Omega} = \left.\dfrac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0. \end{cases}$$

We will talk about recent developments on universal estimates for eigenvalues of the clamped plate problem.