Periods of automorphic functions, Subconvexity of L-functions and Representation Theory

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Abstract

This lecture is based on my joint work with A. Reznikov (see arxiv RT/0504411 and RT/0305351).

Let $\mathbb{H}$ denote the upper half plane equipped with the standard Riemannian metric of constant curvature -1. We denote by dv the associated volume element and by Δ the corresponding Laplace-Beltrami operator on $\mathbb{H}$.

Fix a discrete group Γ of motions of $\mathbb{H}$ and consider the Riemann surface $Y = \Gamma\backslash\mathbb{H}$. For simplicity we assume that Y is compact.

Consider the spectral decomposition of the operator Δ in the space $L^2(Y, dv)$ of functions on Y. It is known that the operator Δ is non-negative and has purely discrete spectrum; we will denote by $0 = \mu_0 < \mu_1 \leq \mu_2 \leq ...$ the eigenvalues of Δ. For these eigenvalues we always use a natural from representation-theoretic point of view parametrization $\mu_i = \frac{1-\lambda_i^2}{4}$, where $\lambda_i \in \mathbb{C}$. We denote by $\phi_i = \phi_{\lambda_i}$ the corresponding eigenfunctions (normalized to have L^2-norm one).

In the theory of automorphic forms, the functions ϕ_{λ_i} are called automorphic functions or *Maass forms* (after H. Maass). The study of Maass forms plays an important role in analytic number theory, analysis and geometry.

In my lecture I will discuss the following problem.

Let us fix two Maass forms ϕ, ϕ', and consider the following *triple product* or *triple period* $c_i = \int_Y \phi\phi'\phi_i$.

I will describe a new method, based on representation theory of the group $SL(2, \mathbf{R})$, which allows to obtain some highly non-trivial estimates for the coefficients c_i as a function of parameter λ_i. I will also discuss how these estimates are connected to some non-trivial estimates of L-functions.