A New Adaptive Variational Model for Liver Segmentation with Region Appearance Propagation

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Abstract  Liver segmentation from Computed Tomography (CT) Angiography images is a crucial step for aiding in liver surgery and in diagnosing liver pathologies. Due to the low contrast, blurred edges, large variability in shape and complex context with clutter features, it is a convoluted problem and still a challenging task. In this paper, we address this problem with an integrated variational model based on the idea of adaptive region growing and region appearance propagation, with which we can focus on the target liver region regardless of the complex but uninterested backgrounds. Our model consists of an edge based term and two novel region based terms which integrate both region intensity and appearance information. An adaptive weight is introduced to spatially balance the intensity region term and the edge term, and at the same time weak liver boundaries are stably delineated by region appearance information. Moreover, the proposed region based model is robust to model parameters, initialization and noise, and it can greatly alleviate the requirement of the scanning protocol and data quality limitation. While segmenting in a slice by slice style often neglects the consecutiveness between slices, we directly segment the liver from the 3D volume data. Last but not least, our model is a nearly automatic one which needs only an arbitrary initial contour/surface inside the liver. Experimental results show that the liver can be accurately and effectively distinguished from the complex backgrounds and vessels can also be simultaneously isolated from the liver with accuracy. Our system is promising for stable practical use and can be also used to segment other abdominal organs.

Keywords  Liver segmentation · Adaptive region growing · Bi-direction force · Region appearance propagation · Complex context · Region based active contour · Vessels segmentation.

Mathematics Subject Classification (2000) 65K10 · 68U10 · 49M30

1 Introduction

As liver disease is one of the most common internal malignancies and also one of the leading death causes, liver intervention becomes one of the most demanding fields in surgery. The treatment of malignant liver diseases targets at the complete destruction or removal of all tumors together with a sufficient safety free margin, at the same time life-critical anatomical structures must be saved. Liver transplantation, the replacement of a diseased liver with a healthy liver allograft, has emerged in recent decades as a critical surgical option for patients with end stage liver disease and acute liver failure. It is also one of the most expensive treatments in modern medicine. Numerous anastomoses and sutures, and many disconnections and reconnections of abdominal and hepatic tissue, must be made for the transplant to succeed. Computerized medical imaging analysis aims at detecting and delineating anatomical structures for surgery planning and diagnosis, which would substantially increase the safety and success rates of surgery. Recently it has gained more attention and has become more and more useful for doctors to make preoperative decisions for liver cancer diagnose and liver transplantation.

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Liver and vessels segmentation from Computed Tomography Angiography (CTA) images are vital for quantitative assessment of treatment options and virtual resection planning and it has been a growing field with open research problems. It should be convenient to provide the radiologist with an automatic segmentation system which takes few time, but segments the liver accurately enough. However, fully automated segmentation is known to be an ill-posed problem due to the fact that there is no clear definition of a correct segmentation without high level prior information. In fact, the fine liver segmentation is still a difficult task. Firstly, low contrast and blurred edges often characterize the CTA images. They are caused by partial volume effects and also heavily influenced by the administration of contrast media, and in different persons, by different machine setup conditions. Secondly, complex context is often the case in the CTA liver images. There is ambiguity of boundaries between the liver and the complex backgrounds: adjacent organs (such as kidney, heart, spleen, stomach, etc) share similar intensities as part of the liver. The complex context also lie in that the backgrounds consist of many abdominal organs and soft tissues with different intensities and shapes. Further difficulties arise from the large variability in appearance, size and shape of liver. These difficulties make most segmentation methods either unfeasible or too complicated to compute.

1.1 Previous Work

There are many approaches for liver segmentation, which can be grouped into two categories:

**Low level information based methods** which consist of intensity based region-growing [24, 16, 17], histogram processing[18, 23], voxel-classification algorithms [12], thresholding[8, 9] with some pre- and post-processing steps. All these methods require an accurate estimate of the liver intensity range. A liver binary volume is created by using the threshold values, after that, some attached neighbor organs and irrelevant tissues are deleted by operations such as morphological operators, holefilling and connected component analysis. Although intensity ranges can be roughly obtained by histogram analysis, but the limitation of these methods is that they need precise threshold values, which is hard to determine. Actually in many cases there are no optimal thresholds. As a result, it is difficult for these methods to isolate the liver effectively without including neighboring tissues with similar intensities. Moreover, they are sensitive to threshold values and can only get coarse liver surface. To get smooth liver surface, active contours have been used to refine the coarse liver surface, e.g., Liu et al. [13] propose a method which combined a GVF snake [28] with edge detectors refine the liver effectively without including neighboring tissues with similar intensities. Moreover, they are sensitive to threshold values, after that, some attached neighbor organs and irrelevant tissues are deleted by operations such as morphological operators, holefilling and connected component analysis. Although intensity ranges can be roughly obtained by histogram analysis, but the limitation of these methods is that they need precise threshold values, which is hard to determine. Actually in many cases there are no optimal thresholds. As a result, it is difficult for these methods to isolate the liver effectively without including neighboring tissues with similar intensities. Moreover, they are sensitive to threshold values and can only get coarse liver surface. To get smooth liver surface, active contours have been used to refine the coarse liver surface, e.g., Liu et al. [13] propose a method which combined a GVF snake [28] with edge detectors refine the coarse liver surface obtained by intensity peak analysis and thresholding.

Besides, several authors have also designed new active contour models to segment the liver, e.g., Pan et al in [3] proposed to explicitly drive an active contour with a dynamic speed function which changes according to the past history of the front. Their approach is to progressively slow the front down as it passes over the boundary points. If the front passes over one boundary point it slows down some. If it passes over two boundary points in sequence, it slows down more, etc. In order to reduce leakage at the liver-rib interface, the propagation is constrained by a-priori anatomic information regarding the distance between liver and skin/ribs. The skin and the ribs are segmented beforehand. This is done through a sequence of operations including thresholding, morphing, and region labeling. However, its intensity range based contour speed is always single direction and it is hard to stop the contour on weak boundaries. Moreover, their model is sensitive to model parameters and the computation complexity is also the weakness. Moreover, previous segmentation of skin/ribs is also not easy.

**Prior information aided methods** such as shape model based methods [7, 10], probabilistic atlas based methods [15] and so on. Prior information can further alleviate the ill-posedness of the segmentation. The exploitation of anatomical knowledge regarding shape, size and position is often used to increase the segmentation performance in many segmentation task. Most of these methods treat segmentation firstly as a statistical estimation problem, while the quality and the support of the training set’s exemplars are often ignored. Building the training set is really a hard work, which needs a lot of training data. While prior geometric model has achieved great success in some segmentation task, several authors have extended it to the liver segmentation task and have shown some promising results. But due to the large variability in appearance, size and shape of liver, model based liver segmentation methods often fail short of accurate segmentation and still an interesting but challenging task.

Most of previous methods segment the liver intuitively in several individual steps with heavy pre-and post-processing and employ different cues in scattered way, which makes the segmentation model less robust and often leads to over-segmentation problem. Besides, with only few pixel level cues, e.g., intensities or gradients, it is impossible to accurately segment the liver from the rest of the anatomical structures. An interesting alternative is to use a unified variational model, with which multi-cues can be integrated simultaneously and it is easy to control the smoothness of the liver surface. However, few variational models have been designed to segment objects with both blurred edges and complex backgrounds. In the present paper, we will investigate in these difficulties and try to solve this complicate liver segmentation problem with a new variational method.
1.2 The Variational Framework and Motivations

A great class of segmentation tasks can be stated in the variational framework through the minimization of a functional, where the solution is given by the evolution equation of an active contour. And usually they can be classified as edge based and region based models. In this direction, two models, namely geodesic active contour model [5] and Chan-Vese(CV) model [6], stand out respectively as the paradigms for edge-based and region-based segmentation methods. Also there is a class of hybrid models that both integrate region information and edge information. A grave drawback is the difficulty to properly balance different terms.

The geodesic active contour (GAC) model is defined as the variational problem

$$\min_C \{E(C) = \int_0^1 g(|\nabla I(C(p))|)|C(p)|dp\},$$

where $I$ is the image which can be 2D and 3D, $C(p)$ is the evolving contour or surface, $g \in (0, 1]$ is the edge detection function that is positive in homogeneous regions, and near zero at the edges

$$g = \frac{1}{1 + \beta |\nabla I|^2},$$

where $\beta$ is an arbitrary positive constant. The gradient descent method gives the flow that minimizes $E(C)$:

$$\frac{\partial C}{\partial t} = (g\kappa - \langle \nabla g, \tilde{N} \rangle)\tilde{N},$$

where $\kappa$ is the curvature of $C$ and $\tilde{N}$ is the inward normal of the contour. The equation (1) is a curve shorten flow and thus we should initialize a contour to enclose the object. Then the contour evolves towards sharp edges, while at the same time acting as a smoothing term. The advantage of this model is the only need of the partial homogeneity of the image, i.e., the homogeneity of regions enclosed by the initial contour and the desired object boundaries. However, this model is only edge based and sensitive to the initialization position, blurred edges and various image artifacts, e.g., spurious edges and noises. Moreover, its convergence speed is very slow and it often hard to capture very concave boundaries. A positive constant balloon force term [1] has been introduced to increase the evolving speed and a sufficiently negative constant will react on the shrinking behavior and force the contour to expand.

$$\frac{\partial C}{\partial t} = (g\kappa + a - \langle \nabla g, \tilde{N} \rangle)\tilde{N}$$

where $a$ is a constant. The GAC model with balloon force is sensitive the choice of the constant $a$.

Comparatively speaking, region based models are more robust to initialization and no edge information is required. In this framework, the image are divided into a number of disjoint regions such that the pixels have high similarity in each region and high contrast between regions. The CV model is based on the piecewise constant assumption and segment the image into two regions. There are many improvements [25, 26] based on local or global statistics in the this paradigm, i.e., the Bayesian generalization of the CV model. All these method model the foreground and background with two different intensity distribution no matter it is parametric or non-parametric. The same idea can be generalized to multi-phase segmentation problems. However, the number of region classes should be predefined and they every region class with an intensity distribution model.

These region based methods are not feasible for liver segmentation. Because the complex backgrounds do not meet homogenate assumption. Although some segmentation tasks with complex backgrounds can be addressed by multi-phase models with great number of region classes, it will cause heavy commontional cost especially for 3D huge volume data. However, what we want is only the liver region which almost has similar appearances or intensities and it is too elaborate to segment all organs. Moreover, large intensity distribution overlapping between the liver and other tissues is existing and segmentation methods relying on intensity homogeneity can not discriminate them well. In fact, these region intensity based methods segment the whole image without any spatial order. On the contrary, the radiologist recognizes the liver mostly relying on edge information. As a result, segmenting with a spatial order as the GAC can help to avoid uninterested background tissues.

In this paper, we propose to conduct the segmentation based on the result of an Information Propagation Process with the GAC model and region growing of continues form as prototypes of this new framework. However, this process is a more adaptive local region. After a contour is initialized inside the object, some reliable features are learned from the initial region. Then the learned information is propagated out until encountering reliable boundaries, which are delineated by these multi feature cues. It is important to note that this new framework is only relying on the homogeneity of the foreground.
Our model is partially inspired by other three models. The first one is the region growing method which examines neighboring pixels of initial seed points and determines whether the pixel neighbors should be added to the region according to some hard threshold values. While the computation is consuming, no matter the time or power, it is sensitive to threshold values and hard to get regular surface. The second one is the hybrid segmentation model proposed by Zhang [20], where they proposed a new region term to locally segment objects with higher intensity values than some global threshold that defined by user manually. Their model is sensitive to the lower threshold. Although he combined this region with an edge term, there is a constant balance between them. As a result, only when a proper threshold is chosen, the two term can balance between each other nearby boundaries. Another underlying assumption is that the surround context must have obviously lower intensity values than the foreground object. Though his model performs well for a class of very simple segmentation tasks, it is not feasible for the complicated liver segmentation. Because several organs adjacent to the liver have similar or higher intensity value than the liver. The third one is the model proposed by Ni [14], where they encourage partitioning the image domain so that the local histograms of intensity within each region are approximately homogeneous. They have applied their model to texture image segmentation problems. But piecewise homogeneity is assumed for this model.

1.3 Overview of Our Method

In this paper, we deal with the problem of the liver segmentation with complex context, blurred boundaries and large intensity overlapping between the liver and adjacent organs. To tackle these problems and the fore mentioned difficulties, we propose a new hybrid variational model based on the idea of adaptive region growing and region appearance propagation in our new Bidirectional Information Propagation framework.

The general idea behind is to firstly initialize a contour/surface inside the liver region and then a combined force based on multi-cues drive the contour out to the right boundary. Meanwhile, in contrast to previous active contour models, reliable intensity and region appearance information learned from initial region are propagated out to help classify new pixels and delineate reliable liver boundaries. As a result, the contour can stop on boundaries where intensity gradient weak but the local intensity or region appearance is locally different. When encountering non-liver organs or tissues no matter what the organs are, the contour will move back and thus further prevent great deal over-segmentation. In this bidirectional propagation manner, we can take full advantage of both the partial homogeneity of the liver image and the most discriminative cues in different regions. As a result, we can focus on the liver region as local as possible and need not to directly build complex model for the whole image.

Another contribution of our model is the spatial adaptive and automatical balance between the region term and boundary term which generally has not received enough attention for most hybrid models. Most models take constant balance parametric for simple segmentation task and have poor performance in complicate tasks, where image properties such as intensity, gradient information, etc, may not be uniformly strong. Actually, the low contrast, blurred edges and complex intensity context are often the case for the contrast enhanced CT liver images and at the same time, most part of liver boundaries have strong gradients. So adaptive balancing weight is more favorable and the region based part of the model and edge based part should dominate the contour evolving in proper regions.

Last but not least, our model can be fast computed and is robust to model parameters, initialization and image artifacts, which are favorable properties for stable practical use.

1.4 Organization of the Work

The paper is organized as follow. In Section 2, we discuss our model in detail and analysis the property of our two new region terms. In Section 3, we briefly discuss the computation of our model and show the procedure to segmentation out both the liver and vessels. Experimental results of tests on some CTA liver images and their analysis are given in Section 3 followed by conclusion in Section 4.

2 Our Proposed Model

2.1 Key Observations for the Liver Segmentation

In spite of complex background, four key observations about the liver region are vital for correct and robust segmentation and also the foundation of our model.

1. Liver boundaries/edges are the most reliable cues to delineate the liver region. However, the edge information, i.e., the gradients, may be not uniformly strong. So boundaries that are weak in intensity gradients but locally prominent should also be captured by our model.
2. The liver is almost homogeneous in intensity and roughly lie in a range \([\mu, \eta]\). It is worth noting that most algorithms in the theory of the CV model, assume the homogeneity of both foreground and backgrounds and little intensity distribution overlapping between them. But neither of the assumptions is the case in the liver segmentation problem.

3. The liver region has similar region appearance which is not pixel level feature and adjacent organs can be further discriminated by region appearances.

4. Image features learned from initial region can be regard as reliable information to constrain the ill-posedness of the liver segmentation. On the contrary, image features learned from training sets are not quite reliable for the large appearance variations in different livers.

To enhance and capture blurred edges, we are relying on two key assumptions. The first one is that, though intensity gradients at blurred edges are weak, most often the local region appearances on both sides are very different. The second is that even though adjacent organs share similar intensities, the region appearances around the weak boundaries are still quite different. Both assumptions have been justified by our experiments and date sets.

While intuitively taking into account multi-cues can result in more robust segmentation, the proper balance of these cues are much more important for correct and robust segmentation. Because image features are non-uniformly distributed, every cue should dominate the segmentation in proper regions.

2.2 Model I: Integrating Edge Information and Region Intensity Information

Here we propose a new hybrid variational model with a novel region based term to softly constrain the intensity of the foreground to a range \([\mu, \eta]\), which is in a spatially adaptive style by varying the influence of thresholds based region term force according to local image information. Our new variational model reads

\[
\min_C \{E_R(C) = \int_0^1 g(|\nabla I|) C'(p) dp + \int_{\Omega_m} w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} d\Omega, \ (3) \]

where \(g(|\nabla I|)\) is the edge detector function defined as Section 1.2, \(\Omega_m\) is the foreground region enclosed by the contour \(C\), \(\mu\) and \(\eta\) are the lower and upper threshold respectively which can be estimated through the analysis of the mean \(m\) and variance \(\sigma\) inside the initial contour and \(w(x)\) is the balancing weight which is spatially varying according to local image properties.

The optimal segmentation minimizing \(E_R(C)\) is computed through evolving an active contour. The corresponding gradient flow equation is

\[
\frac{\partial C}{\partial t} = \{g\kappa - \langle \nabla g, \overrightarrow{N}\rangle + w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} \overrightarrow{N}\}, \quad (4)
\]

where \(\overrightarrow{N}\) and \(\kappa\) are the inward normal and the curvature of the contour \(C\) respectively. Let

\[
v = g\kappa - \langle \nabla g, \overrightarrow{N}\rangle + w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2},
\]

The first two terms in \(v\) correspond to the first term in energy functional \(E_R(C)\) and the third term corresponds to our new region based term. To avoid directly tackling complex backgrounds, a contour or surface will be firstly initialized inside the liver region. Then our new region based force would react on the shrinking behavior of the contour and force it to expand. Pixels on it are classified as the foreground or backgrounds in each step.

**Adaptive Region Growing.** It is obvious that only minimizing the second region based term encourages the contour to enclose the regions with gray-levels lying in \([\mu, \eta]\). In fact, when setting \(w(x) = \text{Constant}\), the second term amounts to continuous form of region growing with predefined accepting range \([\mu, \eta]\). The pixels enclosed by the initial contour can be seen as the seed points. As the evolving of the contour, a pixel on it is accepted as the foreground if the intensity is in \([\mu, \eta]\). However, with a spatial weight \(w(x)\) and the balancing of the GAC term, pixels with intensities lying in \([\mu, \eta]\) may be also classified as backgrounds when \(w(x)\) is small. As a results, our new region term can be regarded as a continues form of **Adaptive Region Growing** term. Similar to the GAC and region growing, this propagation process has favorable local property. However, in this variational framework, this adaptive region growing like process tends to result in more elegant and robust than traditional region growing. Moreover, it can reduce computation cost and result in smooth liver surface.

**The Choice of \(w(x)\).** There are many choices for the adaptive weight \(w(x)\). In the liver segmentation, because the boundary information is the most obvious cue to recognize the liver, it is intuitively reasonable that reliable boundary is the sign to stop the thresholding process. we take the weight to spatially vary according to boundary information and then

\[
w(x) = \alpha g(x) = \frac{\alpha}{1 + \beta |\nabla I|^2}.
\]
where \( \alpha \) and \( \beta \) are arbitrary positive constants. The weight \( w(x) \) goes to zero in regions with great gradients and to \( \alpha \) in homogeneous regions. Other choice such as local intensity variance information, can also be made.

2.3 Property Analysis of Model I

As mentioned before, we first initialize a contour or surface inside the liver region and the new region based force will drive it evolving out. As \( \mathbf{N} \) is the inward normal, the active contour will shrink when \( \upsilon > 0 \) and expand when \( \upsilon < 0 \). As \( \upsilon = 0 \), the contour will stop. It is well known that in the GAC model, the first and second term in \( \upsilon \) dominate the contour in homogenous regions and regions near strong boundaries respectively. The curvature force \( g \kappa \) tends to force the contour to evolve out where the contour is concave and evolve in opposite direction otherwise. The second convection term \( \langle \nabla g, \mathbf{N} \rangle \) is a bi-directional force and will always contract the contour to the boundaries. Let

\[
R := w(x)f, \quad \text{where} \quad f = \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2}.
\]

The third force \( R \) is also bi-directional depending on its sign and adaptive according to intensities and local gradient information. For we have estimated that the intensity in the liver region lies roughly in an intensity range \([\mu, \eta]\), \( R \) will almost be negative (\( R < 0 \)) in the liver region and positive (\( R > 0 \)) for most part of the backgrounds. With a properly chosen balancing weight, \( \upsilon \) will be a expanding force in liver regions and a shrinking one in most background regions. Furthermore, the contour will stop when encountering reliable liver boundaries.

- **Model behavior when the contour is away from boundaries.** Because the gradients inside the liver region or other organs are weak and vary smoothly, \( g \) and \( w(x) \) will be close to one and \( \alpha \) respectively. The second convection term force will be very weak in these regions and at the same time \( R \approx \alpha f \). On the other hand, \( f \) will be close to its negative minimum and then \( R \), when \( I \) is close to the mean of the intensity range, and it will become more positive when \( I \) is far out of the intensity range. As a result, the region based force will be big and dominate the evolving contour when it is away from the boundaries. The contour will quickly evolve out in liver regions and contract when confronting background tissues with intensities out of the intensity range.

- **Model behavior when the contour is nearby boundaries.** When either the intensity is near the threshold bounds (\( f \to 0 \)) or the gradients are sufficiently great (\( w(x) \to 0 \)), the contour/surface will move slowly and even stop. As a result, we can grasp not only sharp boundaries but also blurred edges on the intensity bounds. We analysis this below in two cases:

  **Case 1.** In boundary regions with strong gradients, \( g \) and \( w(x) \) will decay quickly to zero (i.e., \( g \to 0 \) and \( w \to 0 \)). The second term in \( \upsilon \) always attracts the contour to the boundaries. At the same time, as \( f \leq 1 \), \( R = w(x)f \) will tend to be zero and the region based force will be very weak. As a result the GAC term dominate the evolving of the contour in these regions and grasp the great gradients. It worth indicating that even when the intensities outside the boundary fall in to the liver intensity range, great local contrast can also stop the contour. Besides, when intensities outside the boundaries are out of the liver intensity range, the bidirectional region force also act as a boundary attracting force.

  **Case 2.** In blurred boundary regions where gradients are weak, the region based term dominates the evolving contour and will attract it to the boundary, see Fig.1(d). As we can definitely recognize a foreground object from backgrounds when either they are divided by obviously boundaries, which correspond to Case 1 or there are mild gradients and blurred.
edges but outside intensity values are obviously different from the inside intensity values which is the circumstance in this case. As a result, when the contour is outside the boundary, the region term force will be positive and force the contour to contract. When inside the boundary, the region term force is negative and drive the contour to expand. So the bidirectional region term will dominate and find the proper boundary. On the other hand, the second convection term will still weakly attract the contour to the boundary.

So summing above up, the most favorable properties of our hybrid model are that our new region based bidirectional force can tackle blurred boundaries, and the spatially balanced region force and edge force alternately dominate the evolving contour/surface in proper regions. Fig.1 show some difficult cases and our model segment these slices with accuracy. Besides, because of the GAC term and the spatially balancing weight \( w(x) \), the model will also work well with not severely disturbed thresholds, see Fig.5 for a comparison example which shows robustness of our model to the thresholds, i.e., \( \mu \) and \( \eta \).

**Local Adaptive VS Constant Weight for Region Based Model.** An intriguing property of our region based model is the spatially adaptive speed of the contour, which is vital for correct segmentation from complex backgrounds. From the above analysis, in general the contour/surface will evolve quickly inside the liver region despite of image noise and cautiously with fine speed tuned automatically nearby the boundaries. when implementing the model with level set, this property can further prevent leakage problem.

With \( w(x) = \text{Constant} \), the model becomes a simple combination of region growing and the GAC model, which is analogues to Zhang et al’s model \cite{20} and is a generalization to the liver segmentation problem

\[
\min_{C} \{ E_{R}(C) = \int_0^1 g(|\nabla I|) |C'|(p) dp + \alpha \int_{\Omega_{in}} \frac{(I-\mu)(I-\eta)}{(\eta-\mu)^2} d\Omega \}. \tag{5}
\]

The contour will tend to leak out because of poor balance between the region term and edge term. As it is shown in Figure.2, while the adaptive Model I (3) segments this slice perfectly, the contour with equation (5) leaks out both on the right to the inferior vena cava (IVC) and on the left to some soft tissues which sharing the same intensity range as the liver. Moreover the model is very sensitive to thresholds.

![Fig. 2 The effect of \( w(x) \) on segmentation results. (a) The original image. (b) Segmentation results by equation (5) with intensity range \([m - 3\sigma, m + 3.5\sigma]\). (c) Segmentation results by equation (5) with intensity range \([m - 3\sigma, m + 3.5\sigma]\). (d) Segmentation results by our Model I, i.e., equation (3) with intensity range \([m - 3\sigma, m + 3.5\sigma]\).](image-url)

In fact, region based active contour models implemented with level set tend to leak out when there are objects with similar intensity and at the same time they are very close to the liver. Because unlike the traditional snakes, it embeds the contour as the zero level set of a one dimensional higher function. The states of all the image pixels are updated in each step, but traditional contours only update points on the front. This drawback will more serious for models with intensity based region terms, for the intensity include no spatial information. Although with either frequently re-initializing the level set function as sign distance function or narrow band methods we can eliminate this high dimensional effect for pixels far away from the front, for pixels near by the zero level front we need very small time steps to alleviate the leaking out problem which will lead the whole evolving process very slow. On the contrary, its spatially adaptive weight \( w(x) \) amounts to a spatially varying time steps of the contour. In a narrow band of strong boundaries, the region force is very weak and the edge based force dominate the contour and in homogenous region the contour will evolve with full speed. Further stability nearby weak boundaries can obtained with patch level information.

2.4 Model II: Enhancing Model I by Region Appearance Propagation (PAP)

Our proposed Model I is not only simple enough but also effective for a large class of images. However for some cases, its ability and robustness of correct segmentation are limited. The complex context and large intensity distribution
overlapping between different organs often confuse people without anatomical knowledge and thus only pixel level information such as intensities and gradients of intensity can not well discriminate the liver from other tissues. For these cases, we have to tune the parameters carefully and even in some complicate cases we can not get good segmentation results, see Fig.3(b). However, we note that the different organs have different appearance such as intensity statistics, texture distribution and so on, which would help to distinguish them further. To further address the over segmentation problem and the complex context, we propose to employ additional information, e.g., patch level texture, to delineate weak boundaries and control the homogeneity of the liver region. As our previous region term can be seen as a process of Intensity Information Propagation (IIP). Keeping the underlying idea of Model I, we propagate out reliable patch level features learned from initial region. We call this process Region Appearance Propagation (PAP).

In this Region Appearance Propagation framework, we define region appearance as the distribution of some key features, and set the region appearances learned initially as the backbone for correctly segmentation. Then there is great freedom to choose feature descriptors (i.e., features that describes local structures of the image), which can be either point-wise or patch-wise. Suppose the image is \( I(x) : \Omega \rightarrow R \) In order to describe a unified way, we define a feature mapping

\[
F_I : x \in \Omega \mapsto F_I(x) \in R^b
\]

which maps an image \( I(x) \) to its feature space and \( b \) is the dimensional of the feature vector. The simplest choice of \( F_I \) is the identity mapping \( \mathcal{I} \) and \( b = 1 \):

\[
F_I(x) := \mathcal{I}(I(x)) = I(x).
\]

A good choice is the Local Binary Pattern (LBP) which models a specific local structure with a unique integral number. So it is a patch-wise feature but with \( b = 1 \). Other choice can be \( F_I(x) := f(x) \), where \( f \) is defined as Section 2.2 and \( b = 1; F_I(x) := \nabla I(x) \), in which case \( b = 2 \).

The distribution of the feature "signatures" \( F_I(x) \) over an image region provides a discriminative tool to both segment the liver from adjacent organs and control the region appearance consistency of the foreground region. To achieve this, firstly we estimate the rough global liver appearance, i.e., the feature distribution \( P_0 \) of the liver, from the initial region. Secondly, we define the local region appearance around a pixel \( x \in \Omega \) as the distribution of a chosen feature over a neighborhood of it. The region appearance of arbitrary region in the image can be similarly described. Then we compute the similarity \( \mathcal{P}(x) \) between local region appearance of very pixel \( x \) and the global region appearance \( P_0 \). In this manner, we can build a potential field \( \mathcal{P}(x) \) for \( I(x) \) which has high value for low similarity and low value for high similarity.

Probability density function (PDF) can be estimated by parametric or nonparametric method. It is worth noting that estimation of PDF in high-dimensional spaces is notoriously challenging because the available data populates such spaces very sparsely regarded as the curse of dimensionality. Instead, the feature image can be seen as a multichannel image and we model a distribution of one dimension for every channel. Obviously, low dimensional features would result in low computational cost. To illustrate the concept here and reduce computation cost, we set \( F_I(x) = I(x) \) in this paper. However, experimental results beyond this paper have show that LBP is much more favorable but the computation complexity is higher. In addition, we choose histogram as the rough approximate of PDF. Thus each pixel is initially assigned a local estimated histogram (i.e., a normalized histogram of the pixel intensities in a neighborhood of that pixel). Suppose that image \( I : \Omega \rightarrow [0,L] \). Let \( N_x \) be the ball of radius \( r \) centered at \( x \) and \( P_z \) be the local region appearance around \( x \). The corresponding local cumulative distribution function is defined by

\[
F_z(x) := \frac{|z \in N_x \cap \Omega : I(z) \leq y|}{|N_x \cap \Omega|},
\]
for 0 ≤ y ≤ L. The cumulative distribution $F_0$ in the initial region is computed similarly.

To measure the distance of $P_0$ and $P_x$, we select the distance measure introduced by Chan et al [14], i.e., the particular Wasserstein distance as the measure of two PDFs $P$ and $Q$.

$$W(P, Q) = W(F, G) = \int_0^L |F(y) - G(y)| \, dy,$$

where $L$ is the number of the gray levels and $F(y)$ and $G(y)$ is the cumulative distribution functions of $P$ and $Q$ respectively. This distance, which corresponds to a special closed form solution of the Monge-Kantorovich problem, defines a metric and is insensitive to oscillations [27]. So we can build the potential field $\mathcal{P}$ through computing the distance between the predefined liver histogram and the local histogram for every pixel $x$,

$$\mathcal{P}(x) = W(F_0, F_x), \forall x \in \Omega,$$

where $F_0$ is the cumulative histogram of the liver region and estimated from the initial region and $F_x$ is the local cumulative histogram around point $x$. In general, $\mathcal{P}(x)$ tends to be zero inside the liver and has higher values on pixels with different local region appearance. The more different from the liver region the region appearance is, the higher the value of $\mathcal{P}(x)$ gets. So it can be used to further "regularize" the ill-defined liver segmentation problem in these regions and help to control the homogeneity of the liver region. We have noted that the local region histogram can be computed fast by integral images, which is specially useful for high dimensions. Our enhanced energy function reads

$$E_{ER}(C) = \int_0^1 g(|\nabla I|)C'(p) \, dp + \int_{\Omega_m} w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} \, d\Omega + \gamma \int_{\Omega_m} \mathcal{P}(x) \, d\Omega, \quad (6)$$

The corresponding gradient flow equation is

$$\frac{\partial C}{\partial t} = \{g\kappa - \langle \nabla g, \vec{N} \rangle + w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} + \gamma \mathcal{P}(x) \} \vec{N}. \quad (7)$$

As $\mathcal{P}(x)$ is positive and $\vec{N}$ is the inward normal, $\mathcal{P}(x)$ acts as a driven back force.

Fig. 4 Comparison of Segmentation results with model (2), our Model I and Model II. The parameter $\beta$ in $g$ and $w(x)$ is set the same as 0.1. (a) Original image. (b) Segmentation results with balloon force GAC model, i.e., equation (2). (c) Segmentation results with our Model I, i.e., equation (3). (d) Segmentation results with our enhanced Model II, i.e., equation (6).

In our liver segmentation problem, blurred edges is often the case and gradient or intensity based active contours tend to leak out. However, we have observed that although the intensity gradients are weak, the local appearance of boundary points are quite different from that of the liver region. Then $\mathcal{P}(x)$ will have higher values and act as a driven back force to stop the contour, see Fig.3 and Fig.4. As a result, edges are delineate by multi-cues, which is intuitively much more robust.

Moreover, people can also recognize the liver from adjacent tissues when they reveal different region appearances. Analogous resonances explain the performance of our Model II. When confronting tissues with different region appearance, the potential field value $\mathcal{P}$ will increase and driven the contour back. Fig.5 and Fig.8 show some results using the enhanced model. With this region appearance based term we can get more reliable results, especially more reliable vessels segmentation, for example in Fig.6. Moreover, our model becomes much more robust to thresholds, see comparison results in Fig.5. More detail see Section 4.
3 Model Computation and Segmentation Procedures

3.1 Model Computation

Our models can be reformulated into a level set formulation, which handles automatically arbitrary variation in region topology and yield elegant and stable representation of the region membership and boundary. Then Equation (8) can be written as:

\[
\frac{\partial \phi}{\partial t} = \left| \nabla \phi \right| \left[ \nabla \cdot \left( g \frac{\nabla \phi}{\left| \nabla \phi \right|} \right) + w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} + \gamma \rho(x) \right],
\]

where \( \phi \) is the level set function embedding the active contour \( C \). While implicit active contour models avoid several of the difficulties encountered with classical deformable models, the main drawback is its high computational cost. Our model can be solved with an Additive Operator Splitting (AOS) [19] scheme, which is unconditional stable and allows the decomposition of the multidimensional problem into several one-dimensional ones. So our model can be solved quickly even in 3D case.

3.2 Segmentation of the Liver and Vessels

As we have noted that our method can simultaneously segment out the liver without vessels. Let

\[
R_r = \begin{cases} 
1, & x \in r \\
0, & x \in \Omega / r 
\end{cases}
\]

where \( r \in \{ \text{liver}, \text{vessel} \} \). Both the liver \( R_{\text{liver}} \) and vessel \( R_{\text{vessel}} \) can be extracted with ease and accuracy. The procedures of simultaneously segmentation of liver and vessels are as follows:

1. Segment the pure liver region \( R_{\text{pure}} \) without vessels with our proposed hybrid model;
2. Compute the intensity lower bound \( \mu \) of the liver region \( R_{\text{pure}} \);
3. Fill the holes of the liver region \( R_{\text{pure}} \) slice by slice and get the region \( R_{\text{fill}} \). Threshold it with \( \mu \) and then we get the liver region with vessels \( R_{\text{liver}} \);
4. Compute the \( R_{\text{vessel}} \) with

\[
R_{\text{vessel}} = R_{\text{liver}} - R_{\text{pure}}.
\]

4 Results and Discussion

Our methods are validated using images obtained from 12 volume data-sets. Due to the page limit, we show only the segmentation results of several slices from 3 subjects. The data sets used for the evaluation are obtained from the First Affiliated Hospital, Zhejiang University College of Medicine. Every volume consist of about 250 to 450 slices with 512 x 512 resolution for every slice. The algorithm is developed using matlab 2009 and C++ language. To illustrate concepts, we firstly test our model on some difficult 2D slices and the comparisons results of our Model I (3) and Model II (6), the GAC model with balloon force (2) and the simplified version of our model (5). Then we compare our 3D results with the “ground truth” and show some other application.

4.1 Parameters

Suppose that in the initial region, the estimated mean is \( m \) and variance is \( \sigma \). As we have estimated the liver intensity range from part of the liver, a bit wider range \([m - 3\sigma, m + 3.5\sigma]\) is chosen. It is important to note that this choice stays fixed for most cases in our practical use. The parameter \( \beta \) in the edge detector \( g \) is set as 0.1. The stopping criterion is that the total difference of the level set function between the current step and last step is less than \( \varepsilon \), i.e., \( \| \phi^{n+1} - \phi^n \| \leq \varepsilon \), where \( \varepsilon = 0.000005 \) in 2D, and \( \varepsilon = 0.00001 \) in 3D.
4.2 Experiments

For the tested 2D slices, our models, especially Model II, can obtain results as accurate as ground truth. It is worth indicating that, as our model is region based, the initialization can be arbitrary inside the liver. To further reduce the negative effects of initialization to the estimated intensity bounds, we iteratively update a new estimation in the first ten steps. We plot the segmentation results with green contours.

- The performance of our models for complex context segmentation and blurred edges. In Fig.2, some soft tissues are close to the liver and share similar intensities, we compare the our Model I with Model (5) in which a constant balance is chosen. The results show that our Model I can effectively prevent leaking out problems because of its adaptive weight. Surprisingly, our model can even prevent the leakage to the the inferior vena cava (IVC). Fig.3 shows the effect of the region appearance information in tackling blurred boundaries and complex context. Although some soft tissues are glued to the liver and the intensities are also similar, the local feature distribution (i.e., local region appearance) is different. Thus with region appearance information, we can segment it perfectly. In Fig.4, where there are blurred edges near the heart and very weak boundary near the spleen, we compared the performance of the GAC model with balloon force (2) with our models. The GAC model with balloon force require careful parameter tuning and tends to leak our near weak boundaries. Model I can capture a part of weak boundaries, where intensities outside are out of the intensity bounds. However our Model II is superior in capture weak boundaries.

![Fig. 5](image)

**Fig. 5** Comparison of segmentation results of liver vessels by model(2),(3)and (6). More reliable results can be get by our enhanced model (6). (a) The original image. (b) Segmentations with model (2)with finely tuned parameters. (c) Segmentations with our model (3). (d) Segmentations with our enhanced model (6), which can be regarded as ground truth.

- Vessels segmentation. One of the most important aspect is that our method can simultaneously segment out liver vessels with accuracy. Depending on the quality of the original image data, the requirements for vessel analysis methods can be very high, especially in the case of small or closely located vessels, or if the intensity in the vessel lumen or the contrast between the lumen and surrounding structures strongly varies. The thin structure also prohibit many algorithms. While a great class of models penalize high curvature for smoothness, part of the vessels have strikingly high curvature. But by selecting the initial region without liver vessels, we can segment exclude out vessels which will greatly ease the difficulty of vessel segmentation. In Fig.5, we compare our Model I and II with the GAC model with balloon force (2) and show that Model II with patch level region appearance information can distinguish vessels from low contrast images.

- The robustness of our model to model parameters, image noise and image quality. In order to test the sensitiveness of our model to the estimated thresholds values (i.e., $\eta$ and $\mu$), we consider the experiment in Fig.5, where there are soft tissues that are glued to the spleen. These soft tissues have slightly lower intensities. When varying the intensity low bound, Model II can always stably segment the spleen with accuracy, even with $\mu = m - 7.5\sigma$. On the contrary, the simplified version of Model I will quickly leak out and in fact, it is hard to select a proper intensity bounds for this model. Besides, with Model I, we have certain degree of freedom to choose intensity bounds.

- Validation of the accuracy of the segmentation results. Validation of the accuracy of the segmentation results is difficult because ground truth is not available. For comparison, we have to refer to the manual correction by the herpetologist and oncologist as ground truth. Fig.7 shows several difficult 2D slices, where the segmentations are not very accurate, from 3D segmentation results. It is worth noting that we have filled the holes caused by vessels. The results by our model results are plotted with red contours and manual corrected by green contours. By visual inspection, our results are comparable with those produced by expert raters in these difficult cases.

- Segmentation results of other abdominal organs. As above noted that although our model is proposed for liver segmentation, it can be used to segment other organs under similar assumptions. Fig.8 (b) and (b) show the segmentation results of two slices of the spleen. We have noted that the spleen also has neighbor tissues with similar intensity level and our model can segment the spleen accurately. Fig.9 shows the segmentation results of the cholecyst.
Fig. 6 2D segmentation results of spleen with different intensity range. The estimated \( \sigma \) is 15. The results show that our model is insensitive to intensity range. In the first column (a) the original image. (e) The ground truth. In the second column, results by model (5) with intensity range (b) \([m - 3.0\sigma, m + 3.5\sigma]\) and (f) \([m - 4.5\sigma, m + 3.5\sigma]\). In the third column, results by model (3) with intensity range (c) \([m - 3.0\sigma, m + 3.5\sigma]\) and (g) \([m - 4.5\sigma, m + 3.5\sigma]\). In the forth column, results by our model (6) with intensity range (d) \([m - 3.5\sigma, m + 3.5\sigma]\) and (h) \([m - 7.5\sigma, m + 3.5\sigma]\).

Fig. 7 Our model results are plotted with red contours and manual corrected by green contours

5 Conclusions

In this paper, we have formulated the segmentation of liver from complex context as an Information Propagation Process. The GAC model and region growing are shown to be the prototypes of our model. One of the contributions of our work is the formulation of the liver segmentation by an unified variational model which integrates multi-cues, e.g., intensities range, gradients and region appearance. Our model is much more robust to initialization and image quality. To overcome the sensitivity of the estimated intensity bounds and leakage near weak boundaries, we proposed a spatially adaptive weight to balance the intensity based region term and the edge term, and at the same time weak boundaries are enhanced by region appearance information. Furthermore, vessels are distinguish out simultaneously. Experimental results show promising results with our model.
Fig. 8 2D segmentation results of the enhanced model with the fist and second column for liver and third and fourth for spleen. Segmenting the spleen on this case is very convolved for there boundary is two weak. For the enhanced model, $W(F_0, F_1)$ is higher near the weak boundary and the intensity distribution of liver is slightly different from the spleen.

Fig. 9 The segmentation results of the cholecyst.

The main limitation of our model is that, when part of the liver boundaries are totally missed, our low level information based model can not avoid leakage problem. As for future work, we plan to incorporate geometric information to constrain the segmentation.

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References

14. K. Ni , X. Bresson , T. Chan, S. Esedoglu. Local Histogram based Segmentation using the Wasserstein Distance