A New Adaptive Method for 3D Liver Segmentation with Active Contours

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Abstract Liver segmentation is not only the key process for volume computation but also fundamental for further processing to get more anatomy information for individual patient. Due to the low contrast, blurred edges, large variability in shape and complex context with clutter features surrounding the liver that characterize the liver CTA images, it is a convoluted problem and still a challenge task to robustly and accurately segment the liver. In this paper, we overcome these difficulties with a novel variational model based on the idea of intensity probability distribution propagation and region appearance propagation with which we can focus on the target liver regardless of how complex the uninterested background is. Our model consists of an edge based term and two region based term which integrate both region intensity information and region appearance. The model we proposed is robust to model parameters, image noise and greatly alleviate the strict requirement of the scanning protocol and data quality limitation. Moreover, with a spatially varying weight to balance between the region term and edge term, our model can properly switch between the region item force and the edge term force, which makes our model much more robust and perform well for the difficult liver segmentation problem. While segmentation in a slice by slice style often neglects the consecutiveness between slices, we segment the liver from the 3D volume data directly. Experimental results show that our model segments the liver region with accuracy: the liver can be effectively segmented distinguished from the complex background and the vessels are also simultaneously isolated from the liver with accuracy. Our system is promising for stable practical use. Last but not least, our model is a nearly automatic one which needs only an initial contour/surface inside the liver and is also can be used to segment other abdominal organs.

Keywords Liver Segmentation · Adaptive Thresholding · Bi-direction Force · Probability Distribution Propagation · Region Appearance · Complex Context · Region Based Active Contour.

Mathematics Subject Classification (2000) 65K10 · 68U10 · 49M30

1 Introduction

As liver disease is one of the most common internal malignancies and also one of the leading death causes, liver intervention becomes one of the most demanding fields in surgery. The treatment of malignant liver diseases targets at the complete destruction or removal of all tumors together with a sufficient safety free margin, at the same time life-critical anatomical structures must be saved. Liver transplantation, the replacement of a diseased liver with a healthy liver allograft, has emerged in recent decades as a critical surgical option for patients with end stage liver disease and acute liver failure. It is also one of the most expensive treatments in modern medicine. Numerous anastomoses and sutures, and many disconnections and reconnections of abdominal and hepatic tissue, must be made for the transplant to succeed, requiring an eligible recipient and a well-calibrated live or cadaveric donor match. Computerized medical imaging analysis aims at detecting and delineating anatomical structures for surgery planning and diagnosis. For the computer-aided
liver surgery would substantially increase the safety and success rates of surgery, it has gained more attention these years and has become more and more useful for doctors to make preoperative decisions for liver cancer diagnosis and liver transplantation. For example, a reliable volumetric assessment of whole liver and the hepatic segments of potential living donors is one of the key factors in the preoperative donor evaluation.

All these challenges need to tackle a fundamental problem e.g. the extraction of the liver volume from the CTA image data (liver volume segmentation) and it has been a growing field with open research problems. But the fine liver segmentation from CTA scans is still a difficult task. Firstly low contrast and blurred edges often characterize the CTA images. They are caused by partial volume effects resulting from spatial averaging, patient movement, beam hardening and reconstruction artifacts, and also heavily influenced by the administration of contrast media, and in different ones, by different machine setup conditions. Secondly, complex context is often the case in the CTA liver images. There is ambiguity of boundaries between the liver and the complex backgrounds: adjacent organs (e.g. kidney, heart, spleen and stomach) share similar intensities as part of the liver. Segmentation is to extract the liver as foreground while other abdominal tissue is classified as background. The complex context also lie in that their are many other abdominal organs with different intensity and shapes. Further difficulties arise from the large variability in appearance, size and shape of liver which prohibits the using of a prior model. Theses difficulties make most segmentation methods either unsatisfactory or too complex to compute. On the other hand, manual segmentation of the liver is tedious, time consuming, and also lacks of reproducibility. Furthermore, manual segmentation on 2D slices generally neglects the potential impact of the third dimension on the results and often hard to get a regular liver surface. It should be convenient to provide the radiologist with a automatic segmentation system which takes few time, but segment the liver accurately enough. In fact, fully automated segmentation is known to be an ill-posed problem due to the fact that there is no clear definition of a correct segmentation for the computer without high level prior information, especially in complex context. Our aim is to robustly segment organs with complex context such as liver from CT or MR image.

1.1 Related Work

There are many approaches for liver image segmentation, which can be grouped into two categories:

1) Low level information based methods such as gray level statistic based methods, gradient based methods and so on.

Gray level statistic methods which consist of intensity based region-growing [26,16,17], histogram processing[18,25], voxel-classification algorithms [12], thresholding[8,9] with some pre- and post-processing by operations such as morphological operators, holefilling and connected component analysis. Although intensity range of the liver can be roughly got by histogram analysis, but the limitation of these methods is that they need accurately precise threshold values which is hard to get. Further more, often there is no optimal thresholds. So they can’t isolate the liver effectively without including neighboring tissues with similar intensity and they are insensitive to threshold value and often can only get coarse liver surface.

Active contours which is a popular class models that can integrate low level information, has also be used to segment liver. Pan et al in [2] proposed to driven the contour with a dynamic speed function which changes dynamically according to the past history of the front. In addition the propagation is constrained by simple a-priori anatomic information regarding the distance between liver and skin. Liu et al. [13] propose a method which combined a GVF snake with edge detectors and segment liver in a slice by slice style.

2) Prior information aided methods such as model based methods [7,10], probabilistic atlas based methods [15] and so on. Prior information can further constrain the ill-posedness of the segmentation. While Prior geometric model has achieved great success in some segmentation task, Several has extended it to the liver segmentation task and has shown promising result. Most of these methods treat segmentation firstly as a statistical estimation problem, while the quality and the support of the training set’s exemplars are often ignored. Building the training set is really a hard work, which needs a lot of training data. It is worth noting that the exploitation of anatomical knowledge regarding shape, size and position is often used to increase the segmentation performance in many segmentation task. But due to the large variability in appearance, size and shape of liver, a model based liver segmentation is often fall short of accurate segmentation and still a interesting but challenging task.

1.2 the Active Contour Framework and Motivations

Image segmentation defines the process of partitioning an image into several region classes. Most of active contour segmentation models are often stated in a variational framework through the minimization of a functional, where the solution is given by the evolution equation of an active contour. And often they can be classified as either edge based or region based models. Under this framework, two models, namely geodesic active contour model (5) and Chan-Vese(CV)
model [6], stand out respectively as the paradigms for edge-based and region-based segmentation methods. Also there are a class hybrid models that both integrate region information and edge information.

Geodesic active contours (GAC), as an enhanced version of the snake model of Kass et al [11]. The GAC model is defined as the variational problem

\[
\min \{ E(C) = \int_0^1 g(|\nabla I(C(p))|) (C'(p)) dp \},
\]

where \( I \) is the image which can be 2D and 3D, \( C(p) \) is the evolving contour or surface, \( g \in (0, 1] \) is the edge detection function that is positive in homogeneous regions, and near zero at the edges

\[
g = \frac{1}{1 + \beta |\nabla I|^2},
\]

where \( \beta \) is an arbitrary positive constant. The calculus of variations provides the Euler-Lagrange equation of the functional and the gradient descent method gives the flow that minimizes \( E(C) \):

\[
\frac{\partial C}{\partial t} = (g \kappa - \langle \nabla g, \overrightarrow{N} \rangle) \overrightarrow{N}
\]

where \( \kappa \) is the curvature of \( C \) and \( \overrightarrow{N} \) is the inward normal of the contour. This model is good in finding boundaries and regularizing the evolving contour or surface. But this model is only edge based and sensitive to various image artifacts, the initialization step and its convergence speed is very slow. It often hard to capture concave boundaries. Another drawback is that as the contour is to minimize \( E(C) \), the evolving force is single direction and can only shrink in non-boundary regions. So we should initialize the contour outside of the object. Often a positive constant balloon force term[[1]] is used to increase the evolving speed of the GAC contour and a sufficiently negative constant will force the contour to expand.

\[
\frac{\partial C}{\partial t} = (g(\kappa + a) - \langle \nabla g, \overrightarrow{N} \rangle) \overrightarrow{N}
\]

where \( a \) is a constant that act as the balloon force.

We will choose the GAC term as a part of our model to account for boundary attraction force when encountering strong boundary, and boundary regularization. Further we will propose a spatially adaptive bi-direction region force which can drive the contour shrink in some regions and expand in others. The new region term should also capture blurred edges and be robust to image noise, small gradients inside the objects and model parameters.

The CV model with homogeneous region assumption for image \( I \), which is the piecewise constant case of Mumford-Shah model [24].

\[
E(C, c_1, c_2) = \int_0^1 |C'(p)| dp + \lambda \int_{\Omega_1} (I - c_1)^2 d\Omega + \lambda \int_{\Omega_2} (I - c_2)^2 d\Omega,
\]

where \( C \) is the evolving contour, \( c_1 \) and \( c_2 \) are means of the region in and out respectively and \( \Omega \) is the whole image region. The model is based on the homogeneous assumption for every class of region and can be interpreted as segmenting the image by modeling the foreground and background with two Gaussian distribution having the same variance. It builds on the same idea as region competition proposed by Zhu et al [23], which is a statistical and variational framework that is based on minimizing a generalized Bayes and Minimum description length criterion. The model penalizes the boundary length and the Bayes error within each region, in which appropriate probability distributions are chosen. A point in the image will be classified as foreground when its intensity is more likely belong to the foreground distribution. There are many improvements[27,28] based on local or global statistics in the paradigm of region competition. a.e the Bayesian generalization of the CV model. All these method model the foreground and background in the two class segmentation problems with two different intensity distribution no matter parametric or non-parametric. Generalizing to multi-class segmentation problems, these methods need the predefined number of region classes and build a intensity distribution model for every class.

While these model are stable for simple segmentation task with few region classes, they are not feasible for liver segmentation because the complex background does not meet homogenate assumption or has great number of region classes, which will cause heavy computing cost especially for 3D huge volume data. On the other hand, what we want is only the liver which almost has similar region appearance or intensity range. Regions in backgrounds can only be discriminated from the liver either when they are parted by obvious boundaries or when their appearances are different from the liver region, no matter what it is.

In this paper, we shall not follow the philosophy of region competition for the segmentation of liver from complex context and will design two new region terms which will control the homogeny of region appearance such as intensity, texture, etc of the foreground liver region. We segment a pixel into foreground when it is divided by an obvious edge no matter it is sharp or blurred and simultaneously control the region property.
Our model is partially inspired by other two models. The first one is the hybrid segmentation model proposed by Zhang [22], where they proposed a new region term to segment objects with higher intensity value than some threshold that defined by user manually. Their model is sensitive to the lower threshold. Although he combined this region with an edge term, there is a constant balance between them. As a result, only when a proper threshold is chosen, the two term can balance between each other nearby boundaries. Another underlying assumption is that the surround context must have obviously lower intensity value than the foreground object. While his model is performs well for a class segmentation task, it is not feasible for liver segmentation. Because several organs adjacent to the liver have similar or higher intensity value than the liver. The second one is the model proposed by Ni [14] in the philosophy of region competition, where they encourage partitioning the image domain so that the local histograms within each region are approximately homogeneous. They have applied their model to the texture image segmentation problems.

All these models can be reformulated into a variational level set formulation, which handles automatically arbitrary variation in region topology and yield elegant and stable representation of the region membership and boundary. The flexibility of the combination of multi-cues with proper designed region term often leads to robust segmentation results.

Another advantage of the active contour that is vital for our liver segmentation is its favorable local property, as each time it only classify points on the evolving front into background or foreground regardless of faraway foreground points. With the aid of frequently re-initialization, such property can also keep for very faraway points. When encountering objects with similar property and close adjacent to the foreground object, though they are divide by obvious thin boundary, the contour will tend to leak out. In the present paper we will propose a new spatially adaptive region term avoiding such leaking out.

1.3 Overview of Our Method

In this paper, we propose a new hybrid method accounting for the mentioned difficulties with two new region terms for liver segmentation based on the idea of adaptive thresholding and region appearance propagation. Our model is robust to model parameters and image artifacts. The general idea behind is to initialize a contour/surface inside the liver region and then propagate out the region appearance information as the contour evolve out. The contour will stop encountering strong boundary and move back when encountering non-liver organs or tissue no matter what the organ is. In this manner we need not to directly build complex model for the whole image.

Our new model is based on the following two observations. The fist key observation is the almost intensity homogeneity in the liver region. We propose an threshold based adaptive region based term which only needs rough intensity range that can be estimated automatically from initial region. At the same time we must not segment adjacent organs sharing similar intensity with the liver. The second observation is that the liver region has similar region appearance or sometimes similar texture which is not pixel level feature. For over-segmentation is the potential problem in the segmentation, we want to use higher level appearance information to constrain the segmentation process. We are relying on two key assumptions. The fist is that, though gradients at blurred edges are weak, the local region appearance is very different from the inside liver region. The second is that when adjacent organ shares the similar intensity, there region appearance maybe quite different. We model local appearance by local probability distribution or histogram, with which we can constrain texture and the density distribution of the foreground liver region.

We propose to take this complex context liver segmentation problem as a combination of an adaptive thresholding process which is actually an adaptive intensity information propagation process and a probability distribution (PD) propagation process as a special case of region appearance propagation. Based on this idea, if we can have the reliable PD for the seed region, we can propagation out the PD and accept new points with similar local PD. The propagation process will stop with the aid of the first region term when confronting different region with much different PD. So we need not to build model for the complex context. It is important to note that this new framework actually segments based on local appearance contrast and it can integrate more cues for similar segmentation problem such as gradient information, illuminate variance and so on.

Another contribution of our model is the spatially and automatically balance between the region term and boundary term which generally has not received enough attention for most hybrid models. While most models take constant balance parametric for simple segmentation task and has poor performance in complex task. Adaptive balance will perform well, because image properties such as intensity, gradient information etc may not be uniformly strong. The low contrast and blurred edges are often the case for the contrast enhanced CT liver images and due to complex context, neighbor organs often have similar intensity but with local contrast. So proper part of the model should dominate in proper spatial regions. Furthermore, our model can avoid several kind of leaking out problem that will be analysis in detail later.
1.4 Organization of the Work

The paper is organized as follow. In section 2, we discuss our model in detail and analysis the property of our two new region term. In section 3 we briefly discuss the computation of our model and show the procedure to segmentation out both the liver and vessels. Experimental results of tests on the CTA images and their analysis are given in Section 3 followed by conclusion in Section 4.

2 Our Proposed Model

While active contour /surface using level set is a flexible framework for medical segmentations by taking into account multi-cues for correct segmentation, the CTA liver images which are contrast enhanced image, exhibit many special characters that our segmentation model can benefit from. Edge information is the most important cue for correct segmentation in CTA. As the liver region can be recognized from adjacent organ only when there are obvious boundary. But these edge information, a.e the gradient may not uniformly strong. Some regions there are strong gradients while somewhere gradients may not be strong enough and even very weak. In this paper we would like that our model will grasp both strong and blurred boundaries information. Furthermore the model should spatially balance between region and edge information to account for non-uniform distributed image properties.

2.1 Model Integrating Region Intensity information and Edge Information

We have observed that the intensity of the liver region, in the contrast enhanced CTA image, roughly lie in a range $[\mu, \eta]$, but this intensity range is different for images of different individuals and images produced by different CT machines. Given this observation, it is intuitively that taking thresholds and threshold based region growing method seem good. But they often suffer many problems. With hard thresholds, good thresholds often distinguish most part of the liver region, but we often can’t get the exact thresholds and perfectly segment whole liver without irrelevant tissue, as their is often no proper global thresholds for the liver images. It is time consuming to manually select proper thresholds for every individual image, while automatic estimation often caused low inaccurate results with these segmentation methods. Further it is hard to get a smooth and sub-pixel accurate liver surface. Here we can tackle these problems based on the idea of soft/adaptive thresholding in our active contour framework relying on its local property and flexibility to integrate multi-cues and further alleviate it based on the idea of probability propagation later on. In this paper, we propose a novel region based term to softly constrain the intensity to a range $[\mu, \eta]$ in an spatially adaptive style by varying the influence of thresholding like region term force in different regions regarding to local image information, while simultaneously balancing with the edge based GAC term. Our new variational model with a new region term reads

$$E_R(C) = \int_0^1 g(|\nabla I|)|C'(p)|dp + \alpha \int_{\Omega_R} g(|\nabla I|)\frac{(I-\mu)(I-\eta)}{(\eta-\mu)^2}d\Omega,$$

with $\mu$ and $\eta$ are the lower and upper threshold respectively which can be computed by the analysis of the mean $m$ and variance $\sigma$ of the initial region enclosed by the initial contour; $g$ defined as before is to spatially balance the region and edge term according to boundary information; $\alpha$ is a constant balancing weight. It is obvious that minimizing the second region based term encourages the contours to enclose the regions with gray-levels lying in $[\mu, \eta]$. But in regions with great gradients, $g$ will tends to zero and then the region term integral in these regions will contribute less to the energy functional $E_R(C)$. So it is can be viewed as an adaptive thresholds based term. The corresponding gradient flow equation is

$$\frac{\partial C}{\partial t} = \nu \vec{N} = \{g\kappa - \langle \nabla g, \vec{N} \rangle + g\frac{(I-\mu)(I-\eta)}{(\eta-\mu)^2}\} \vec{N},$$

$$\nu = \{g\kappa - \langle \nabla g, \vec{N} \rangle + \alpha g\frac{(I-\mu)(I-\eta)}{(\eta-\mu)^2}\}.$$
2.2 Model Property Analysis

Let
\[ R := \alpha g f, 
\]
\[ f := \frac{(I - \mu)(\eta - I)}{(\eta - \mu)^2}, \]

\[ R \] is a bi-direction region based force depending its sign and adaptive according to local gradient information controlled by \( g \). As \( \vec{N} \) is the inward normal. The active contour will shrink when \( v > 0 \) and expand when \( v < 0 \). As \( v = 0 \), the contour will stop.

It is well known in GAC model, the force \( g \kappa \) tends to force the contour to evolve out where the contour is concave and evolve in otherwise. Nearby the boundary, the second convection term \( \langle \nabla g, N \rangle \) as a bi-direction force will always contract the contour to the boundary. The third force \( g \frac{(I - \mu)(\eta - I)}{(\eta - \mu)^2} \) corresponding to our new region term is also a region based bi-direction force and at the same time aware of image gradients. For we have estimated that the intensity in the liver region lies roughly in an intensity range \([\mu, \eta]\), \( R \) will almost be positive \((R > 0)\) in the liver region and negative \((R < 0)\) for most part of the backgrounds. For organs share the similar intensity, \( R \) will also positive. With a properly chosen balancing weight, \( v \) will be positive in liver regions and negative in most background regions. Further more the contour should stop encountering liver boundaries.

In contrary to most of the active contour models, firstly we initialize a contour or surface inside the liver region. Because the gradients inside the liver region or other organs are weak, \( g \) will tends to one and nearly constant. The second convection term will tend to zero in these regions and at the same time \( R \approx \alpha f \). On the other hand, \( f \) will take its positive maximum when \( I \) is equal to the mean of the intensity range and become more negative when \( I \) is far out of the intensity range. As a result, the region based force will dominate when away from the boundaries and the contour will quickly evolve out in liver regions and contract when confronting background tissues with different intensity range. But when either the intensity near the threshold values \( f \to 0 \) or the gradient is sufficiently great \( g \to 0 \), the contour/surface will moving slowly and even stop. We will analysis it in two cases:

Case 1. In boundary regions with strong gradient, \( g \) will decay quickly to zero, a.e \( g \to 0 \). it is well known that at both sides of the boundary, the second term will always attract the contour to the boundary. At the same time, as \( F \leq 1 \), \( R = \alpha g F \) will tend to zero and the region based force will faint. As a result the GAC term dominate the evolving of the contour in these regions.

Case 2. In boundary regions with blurred edges where gradients are mild, the region based term dominates and will attract the contour to the boundary. As we can definitely recognize a foreground object from backgrounds when either they are divided by obviously boundary that correspond to Case 1 or there is smoothly varying intensity but outside intensity value is obviously different from the inside intensity value that the circumstance in this case. As a result, when the contour is outside the boundary the region term force will be negative and force the contour to contract. While inside the boundary, the force is positive and force the the contour expand. So the bi-direction region term will dominate and find the proper boundary. Moreover the second convection term will still weakly attract the contour to the boundary.

So summing above up, one of most favorable properties of our hybrid model is that the simultaneously balanced region term force and GAC term force alternately dominate the evolving contour/surface in proper regions. Figure show some difficult cases and our model segment these slices with accuracy, more details and examples see Section 4. Because the GAC term and the spatially balancing weight \( g \), the model will also work well for not severely disturbed thresholds, see figure.3 for a comparison which shows that our model is robust to the thresholds, a.e \( \mu \) and \( \eta \).
Another intriguing property of our model is that the contour/surface will evolve with an adaptive speed, which is vital for correct segmentation. From the above, in general the contour/surface will evolve quickly inside the liver region despite of image noise and cautiously with fine speed tuned automatically nearby the boundaries. As mentioned in Section 1.2, most active contour models implanted with level set method tend to leak out when there are objects with similar intensity but divided by a thin boundaries. As unlike the pure snake, it embed the contour as the zero level set of a one dimensional higher function. The sates of the whole image are updated in each step while traditional contours only update points on the front. This drawback will more tedious for models with intensity based region terms. Although with either frequently re-initializing the level set function as sign distance function or narrow band methods we will eliminate this high dimensional effect for point far away from the front, the leaking out is only be avoid with either very small time step which will lead the whole evolving process very slow. On the contrary, our model will not leak out because of its spatially adaptive weight g, which amounts to a spatially varying time step, see Fig.2(b) for some comparison examples about segmenting the spleen a convolved slice.

Without g, a pure threshold based region term which is a generalization of Zhang et al’s model [22] for liver segmentation

$$E_R(C) = \int_0^1 g(|\nabla I|)|C'(p)| dp + \alpha \int_{\Omega_m} \frac{(I-\mu)(I-\eta)}{(\eta-\mu)^2} d\Omega, \quad (5)$$

will not work well, the contour will leak out quickly for poor balance between the region term and edge term. As it is shown in Figure.2, while the adaptive model a.e equation (3) segments this slice image perfectly, the contour with equation (5) leaks out both on the right to the inferior vena cava (IVC) and the the left to some soft tissue sharing the similar intensity. Moreover the model is very sensitive to thresholds.

Fig. 2 The the effect of g to segmentation results. (a) The original image; (b) Segmentation results by equation (3) with intensity range $[m - 3\sigma, m + 3.5\sigma]$; (c) Segmentation results without g.a.e equation (5) with $[m - 3\sigma, m + 3.5\sigma]$; (d) Segmentation results without g.a.e equation 5 with $[m - 2\sigma, m + 3.5\sigma]$;

Fig. 3 2D segmentation results of spleen with different intensity range with our model, equation (3). All the segmentations are with the same initialization and the estimated $\sigma$ is 15.6. The results show that our model is insensitive to the estimated intensity range. (a) the original image; (b) with intensity range $[m - 1.5\sigma, m + 3.5\sigma]$; (c) with intensity range $[m - 2.5\sigma, m + 3.5\sigma]$; (d) with intensity range $[m - 5.5\sigma, m + 3.5\sigma]$.

Therefore with this hybrid active contour model that has good local property, we can segment the focus organ locally, regardless of the complex context, by propagating out the intensity range information learned from the initial region. Further the spatially adaptive weight make our active contour evolving in an adaptive style and also make our model robust to model parameters.
2.3 Enhance Our Model by Region Appearance Propagation (PAP)

Our proposed model is not only simple enough but also effective for a large class of images. But for some cases, its ability of correct segmentation is limited. The complex context and the similar gray level between different organs often confuse people without anatomical knowledge. While only pixel level information such as gray level intensity information and gradient information can not well discriminate the liver region from others. In this case, we have to tune the parameters carefully and in some complex we can not get good segmentation results, see Fig.4(b). We note that the different organs have different global appearances or textures, which shall help to discriminate them further. To further integrate local appearance and keep the underlying idea of our previous region term for tackling complex context, we enhance our model with a new region term to keep the region appearance of the liver consistent in the process of the active contour evolution. We call this process Region Appearance Propagation (PAP). On the contrary, our previous region term can account for a process of Intensity Information Propagation (IIP).

In this Region Appearance Propagation framework, we have great freedom to choose region appearance descriptors. Here we choose to describe region appearance by global and local statistics. Given the initial region, we can roughly estimate the global probability distribution of the liver region by estimating the probability distribution in the initial region. We hope that the segmented region has similar probability distribution. The method we adopt is to propagate out the initial probability distribution information, which can also be updated iteratively, as the expansion of the evolving contour. While points with similar local probability distribution will be classed to the liver tissue, the points on the evolving fronts with deferent local appearance may have high probability to be excluded. In this manner, predefined region appearance information will propagate out and further prevent the evolving contour/surface from leaking out. We call this method Probability Distribution Propagation (PDP) as a special case of RAP.

Probability density function (PDF) can be estimated by parametric and nonparametric method. Histogram can be used as a rough approximate of probability density function and its computation cost is low. Each pixel is initially assigned a local estimated histogram, i.e. a normalized histogram of the pixel intensities in a neighborhood of that pixel. Suppose that image \( I : \Omega \rightarrow [0, L] \). Let \( N_{cr} \) be the ball of radius \( r \) centered at \( x \). The local cumulative distribution function is defined by

\[
F_c(y) := \frac{| z \in N_{cr} \cap \Omega : I(z) \leq y |}{| N_{cr} \cap \Omega |},
\]

for \( 0 \leq y \leq L \).

The model finds a partition such that the local histograms in the liver region are similar. So we can build a potential field \( \mathcal{P} \) through computing the distance between the predefined liver histogram and the local histogram for every point. The potential field has low value in the liver region and high value in regions with different appearance and further more its value at boundary point are slightly high. So it shall further “regularize” the ill-defined liver segmentation problem in these regions.

Assume that the most of the liver region have the PDF \( F_0 \) with the cumulative distribution function \( F_0 \), the pixels on the front of the evolving surface will be seen belong to the liver in higher probability when its local histogram is more similar to \( F_0 \). When confronting points with different probability distribution, the contour/surface stops or slows down. We select the distance introduced by Chan et al [14], i.e. the particular Wasserstein distance as the distance measure of two PDFs \( P \) and \( Q \).

\[
W(P, Q) = W(F, G) = \int_0^L |F(y) - G(y)|dy,
\]

where \( L \) is the number of the gray levels. \( F(y) \) and \( G(y) \) are the cumulative distribution functions of \( P \) and \( Q \) respectively. This distance, which corresponds to a special closed form solution of the Monge-Kantorovich problem, defines a metric and is insensitive to oscillations [29]. As a result

\[
\mathcal{P}(x) = W(F_0, F_c).
\]

Our enhanced model energy function reads

\[
E_{ER}(C) = \int_0^1 g(|\nabla I|)C(p)dp + \alpha \int_{\Omega_n} g(|\nabla I|) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} d\Omega + \gamma \int_{\Omega_n} W(F_0, F_c) d\Omega. \tag{6}
\]

where \( F_0 \) is the cumulative histogram of the liver region and is often estimated from the initial region histogram and \( F_c \) is the local histogram at point \( x \). We have noted that the local region histogram can be computed fast by integral images, which is specially useful for high dimensions. The corresponding gradient flow equation is

\[
\frac{\partial C}{\partial t} = \{ g\kappa - \langle \nabla g, \vec{N} \rangle + g \frac{(I - \mu)(\eta - I)}{(\eta - \mu)^2} - \mathcal{P}(x) \} \vec{N}. \tag{7}
\]
As $W(F_0, F_x)$ is positive and $\mathcal{N}$ is the inward normal, $\mathcal{P}(x)$ act as driven back force. In general, $\mathcal{P}(x)$ tends to zero inside the liver and increase in points with different local region appearance. The region appearance is more different from the liver region, the higher the value of $\mathcal{P}(x)$ gets.

In our liver segmentation problem, blurred edges is often the case and gradient based active contours tend to leak out. While our previous region term can solve this problem to a certain degree, for weak boundaries the performance is still unstable. But we have observed that although the gradient is weak, the local appearance of boundary point are quite different from the liver region. Then $\mathcal{P}(x)$ will tends to become high and act as a driven back force to stop the contour, see ure.6 (g)(h) as an example.

Moreover as we only recognize the liver from adjacent tissues when there are in different region appearance. As our region appearance descriptor is PDF, both intensity and texture information are integrated. When confronting tissues with different region appearance and so with high potential field value $\mathcal{P}$, the contour will further be constrain to the liver region.

Note that we have did not employ the popular idea of the region competition because of the complex context, but we evolve the surface/contour based on the idea of probability propagation, which kicks out the problem of the complex background. In this manner, we only need build the probability distribution for the liver region and propagate it out regardless of the complex context where have complex probability distribution. Fig.5 and Fig.6 show some results using the enhanced model. With this region appearance based term we can get more reliable results, for example in Fig.4, and our model will become more robust to thresholds, see comparison results in Fig.5.

3 Model Computation and Liver and Vessels Segmentation

Equation (8) can be written in the level set form:

$$\frac{\partial \phi}{\partial t} = |\nabla \phi| \left[ \nabla \left( g_1 \frac{\nabla \phi}{|\nabla \phi|} \right) + \alpha g_2 (I - \mu)(\eta - I) \left( \frac{\eta - \mu}{\eta - \mu} \right) - \gamma W(F_0, F_x) \right].$$

(8)

$\phi$ is the level set function embedding the active contour $C$. While implicit active contour models avoid several of the difficulties encountered with classical deformable models, their main drawback is computational cost. Our model can be solved with an Operator Splitting Scheme (AOS) [21], which is unconditional stable and allows the decompostion of the multidimensional problem into several one-dimensional ones. So our model can be solved quickly even in 3D case.

A classical discretization of the our region based terms can lead to the creation of loops. The entropy condition of Sethian [1] and [1] can prevent the curve from propagating where it has already been and thus avoids loop formation.

As we have noted that our method an simultaneously segment out the liver without vessel. As a result both the liver $R_{liver}$ and vessel $R_{vessel}$ can be extract with ease and accuracy. The procedures of simultaneously segmentation of liver and vessels are as follow:

1. Segment the pure liver region $R_{pure}$ without vessels with our proposed hybrid model, a.e equation (3) or equation (6);
2. Compute the intensity lower bound $\mu$ of the liver liver region $R_{pure}$ which is segmented in last step.
3. Fill the holes of the liver region $R_{pure}$ slice by slice and get the region $R_{fill}$ and threshold it with $\mu$. Then we get the liver region with vessels $R_{liver}$.
4. Compute the $R_{vessel}$ with

$$R_{vessel} = R_{liver} - R_{pure}.$$

![Fig. 4](image)

Fig. 4 With the enhanced model, a.e equation 6, we get more reliable results. (a) The original image; (b) Segmentation results by equation 2 with intensity range $[m - 3\sigma, m + 3.5\sigma]$; (c) Segmentation results without equation 6 with $[m - 3\sigma, m + 3.5\sigma]$. 
Fig. 5 2D segmentation results of spleen with different intensity range. The estimated $\sigma$ is 15. The results show that our model is insensitive intensity range (a) the original image; (b) with intensity range $[m - 2.5\sigma, m + 3.5\sigma]$; (c) with intensity range $[m - 5.5\sigma, m + 3.5\sigma]$; (d) with intensity range $[m - 7.5\sigma, m + 3.5\sigma]$.

Fig. 6 2D segmentation results of the enhanced model with the first and second column for liver and third and fourth for spleen. Segmenting the spleen on this case is very convoluted for there boundary is two weak. For the enhanced model, $W(F_0, F_1)$ is higher near the weak boundary and the intensity distribution of liver is slightly different from the spleen.

4 Results and Discussion

Our method has been validated using images obtained from 36 volume data-sets. Due to the page limit, we show only the segmentation results of several slices from 3 subjects in Fig. 5. The datasets used for the evaluation are obtained from the First Affiliated Hospital, Zhejiang University College of Medicine. They consist of about 250 to 450 slices with $512 \times 512$ resolution for every person. The algorithm has been developed using matlab 2009 and C++ language. We conducted three experiments for visual analysis to confirm the our new model. Suppose that the estimated mean $m$ and variance in the initial region is $\sigma$. The intensity range can be chosen as $[m - 3\sigma, m + 3.5\sigma]$. Figure.7 show some key slices from three data sets.

Firstly we test our model in several 2D cases. The region based speed function $R$ adapts with the gradient is vital for our liver segmentation, because the liver is often surrounded by organs with similar gray level intensity. Fig.2(a)-(f) show some segmentation results with tissues having similar intensity around the liver. Another important aspect is the adaptive evolving speed will also account for adaptive step size in implementation, which is important for preventing leaking out. For example in Fig.2(a), although near the boundary between the right kidney and the liver, the kidney exhibits similar gray level as the part of the liver, locally the kidney is brighter than the liver. Without $g$, the region speed $R$ will positive and large near the boundary and thereby the contour will evolve into the kidney quickly. But with $g$, the speed $R$ will be small due to great gradients and the boundary based term GAC will dominate the evolving. In Fig.2(f), there is a small non-liver region that have similar gray level as the liver, but they are divided by a thin dark line. The contour/surface...
Fig. 7 Several key slices of segmentation results by our model for three data sets. (a) and (b) for one person; (c) and (d) for the other two respectively. They show our algorithm are accurate.

tends to leak out in these region in implementation. We need smaller step size for these regions to prevent leaking out, but our speed function $R$ tend to zero as there are great gradients and the boundary term will dominate in these regions.

In order to test the sensitteness of our model to the estimated thresholds values, a.e $\eta$ and $\mu$, we consider the experiment in Fig.3, and vary the value of $\eta$. The 2D segmentation results of the spleen show the robustness of our model. Further we test our enhanced model, see Fig.5, which show more robustness of our model. Figure.2 shows the importance of the spatially gradient aware weight which is vital for the convolved live segmentation In Fig.4, we compared the usefulness of integrating high level region appearance information in tackling blur boundaries and complex context surround the liver.

Validation of the accuracy of the segmentation results is difficult because ground truth is not available. For comparison, we have to refer to the manual correction by the herpetologist and oncologist. The manually segmented images are used as ground truth. Fig.6 shows the experimental results for different slices liver CT images. The results by our model results are plotted with red contours and manual corrected by green contours. By visual inspection, our results are comparable with those produced by expert raters.

Fig. 8 Our model results are plotted with red contours and manual corrected by green contours.

As above noted that although our model is proposed for liver segmentation, it can be used to segmentation other organs that can be made similar assumption. Fig.9 show some of the results for other medical applications. One of the most important aspect is that our method can simultaneously segment out liver vessels with accuracy, see Figure.7, even for very noise images such as Fig.6(e). Depending on the quality of the original image data, the requirements for vessel analysis methods can be very high, especially in the case of small or closely located vessels, or if the intensity in the vessel lumen or the contrast between the lumen and surrounding structures strongly varies. The thin structure also prohibit many algorithm. While a great class of models penalize high curvature for smoothness, part of the vessels have strikingly high curvature. But by selecting the initial region with less or without liver vessel so we can segment exclude
out vessels which will greatly ease the difficulty of vessel segmentation. Actually we can several steps of simple low level postprocess will segmentation exactly most vessels. Fig.6 (g) and (h) show the segmentation results of two slices of the spleen. We have noted that the spleen also have neighbor tissue with similar intensity level and our model can segment the spleen accurately. Fig.9 (c) and (d) show the segmentation results of the cholecyst.

5 Conclusions

In this paper, we proposed an robust segmentation algorithm for liver segmentation with two new region terms. Our approach is an hybrid method on the idea of a adaptive threshold and probability distribution propagation and has spatially simultaneous balancing between region based term and boundary based term. Thanks for the level set method, our model can both segment slice by slice and directly in 3D case which is more reliable for liver segmentation. Moreover, since our energy is defined by a continuous setting , we can get a accurate smooth liver surface and it is easy for volume computing. Our method can be further improve by integrating more reliable region appearance descriptors. The threshold values can also be updated in first few steps until it get stable. We have also built a software which is easy to use with many useful tools usable for 2D and 3D segmentation problems. In experimental results we showed that accurate segmentations of different quality date sets can be obtained in less than five minutes for our big volume.

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