Basic Hopf Algebras Of Finite Representation Type And Their Classification *

Gong-xiang Liu† Fang Li‡
Department of Mathematics
Zhejiang University
Hangzhou, Zhejiang 310028, China

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Abstract

Let k be a algebraically closed field. We give a necessary and sufficient condition for a basic Hopf algebra over k to be finite representation type. Explicitly, we prove that a basic Hopf algebra over k is finite representation type if and only if it is Nakayama. By this conclusion, we classify all basic Hopf algebras over k of finite representation type.

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1 Introduction

In this paper, let k be an algebraically closed field and all spaces are k-spaces.

In the representation theory of algebras, one remarkable conclusion, due to P.Gabriel, states that for any basic algebra A over k, there exists a unique quiver Γ_A such that $k\Gamma_A/I \cong A$ as algebras, where $J^N \subseteq I \subseteq J^2$ $(N \geq 2)$ and J is the ideal generated by all arrows. An advantage for this conclusion is that we can transform the study of A-modules to that of representations of path algebra with relation (see [1]).

Our aim is to characterize finite-dimensional basic Hopf algebras of finite representation type by using above method. As a special kind of basic algebras, there must be some additional restrictions on the corresponding quiver on a basic Hopf algebra. Fortunately, Green and Solberg proved that this corresponding quiver must be a so-called covering quivers (see [6]). This fact is important for us.

*Project(No.102028) supported by the Natural Science Foundation of Zhejiang Province of China
†gxliu7969@yahoo.com.cn
‡fangli@zju.edu.cn

At first, we give the definition of covering quivers. As a byproduct, we discuss the relationship between covering quivers and Hopf quivers which was defined in [3]. It turns out that they are equivalent (see Proposition 2.1). Some results from [6], which are needed in this paper, are introduced as preliminaries.

In section 3, we will give the main result, that is, a finite-dimensional basic Hopf algebra over k is finite representation type if and only if it is Nakayama (Theorem 3.1). In the view point of representation theory, Nakayama algebras is the best understood artin algebras next to semisimple algebras and their Auslander-Reiten quivers are given clearly (see [1]). Thus we one can see that we can draw Gabriel quivers and Auslander-Reiten quivers of basic Hopf algebras of finite representation type directly.

By a conclusion in [2], we will show that a finite-dimensional basic Hopf algebra over k is finite representation type if and only if it is monomial (see [2]). In [2], the authors described the Auslander-Reiten quivers of monomial Hopf algebras. From this, we can deduce that all finite-dimensional monomial Hopf algebras are finite representation type. Thus our conclusion implies that they are precisely all basic Hopf algebra of finite representation type.

The authors of [2] classified all monomial Hopf algebras when the characteristic of k is zero. In [7], the first author of this paper and Y.Ye have given a description of the structures of monomial Hopf algebras when the characteristic of k is not zero. By these conclusions, we give a classification of basic Hopf algebra of finite representation type in Section 4.

2 Preliminaries

This section will relay heavily on two beautiful papers [5][6] and the book [1].

Quivers considered here are always finite. Given a quiver $\Gamma = (\Gamma_0, \Gamma_1)$ with Γ_0 the set of vertices and Γ_1 the set of arrows, denote $k\Gamma$ the path algebra of Γ. For $\alpha \in \Gamma_1$, let $s(\alpha)$ and $t(\alpha)$ denote respectively the starting and ending vertex of α. An ideal I of $k\Gamma$ is *admissible* if $J^N \subseteq I \subseteq J^2$ for some positive integer $N \geq 2$, where J is the ideal generated by all arrows.

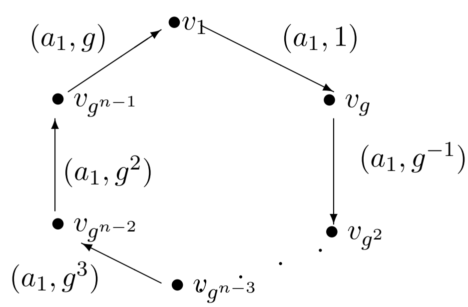
For any finite dimensional algebra Λ, we denote the Jacobson radical of Λ by J_Λ. Λ is said to be *basic* if $\Lambda \cong \oplus_{i=1}^n P_i$ for some indecomposable projective Λ-modules P_i, then $P_i \ncong P_j$ for $i \neq j$. It is known that a basic algebra Λ over an algebraically close field k is *elementary*, i.e. $\Lambda/J_\Lambda \cong k \times k \times \cdots \times k$. A remarkable conclusion in representation theory, due to P.Gabriel, states that, for any elementary algebra Λ, there exists a unique finite quiver Γ and an admissible ideal I of $k\Gamma$, such that $\Lambda \cong k\Gamma/I$ (see [1]).

Next, let us recall the definition of *covering quivers*(see [6]). Let G be a finite group and let $W = (w_1, w_2, \ldots, w_n)$ be a sequence of elements of G. We say W is a *weight sequence* if, for each $g \in G$, the sequences W and $(gw_1g^{-1}, gw_2g^{-1}, \ldots, gw_ng^{-1})$ are same up to a permutation. In particular, W is closed under conjugation. Define a quiver, denoted by $\Gamma_G(W)$, as follows. The vertices of $\Gamma_G(W)$ is the set $\{v_g\}_{g\in G}$ and the arrows are given by

$$\{(a_i, g) :\ v_{g^{-1}} \to v_{w_i g^{-1}} | i = 1, 2, \ldots, n, g \in G\}$$

We call this quiver the covering quiver (with respect to W).

Example 2.1 *(1): Let $G =< g >$, $g^n = 1$ and $W = (g)$, then the corresponding covering quiver is (We call such quiver a* basic cycle of length n*)*



(2): Let $G = K_4 = \{1, a, b, ab\}$, the Klein four group, and $W = (1)$. Then the corresponding covering quiver is

$$\bullet^1 \circlearrowleft, \quad \bullet^a \circlearrowleft, \quad \bullet^b \circlearrowleft, \quad \bullet^{ab} \circlearrowleft$$

At present, we digress to discuss the relationship between covering quivers and *Hopf quivers* which defined in [3]. Let us recall it.

Let G be a finite group and $\mathcal{C}$ the set of conjugate classes. Denote the set of natural numbers by $\mathcal{N}$. A class function $\chi : \mathcal{C} \to \mathcal{N}$ is called a *ramification*, and denoted by $\chi = \sum_{C\in\mathcal{C}} \chi_C C$. Given a ramification $\chi = \sum_{C\in\mathcal{C}} \chi_C C$ of G, then corresponding Hopf quiver $\Gamma(G, \chi)$ has the set of vertices $\Gamma_0 = G$, and for each $x \in \Gamma_0, c \in C \in \mathcal{C}$, one has χ_C arrows from x to cx.

Given a covering quiver $\Gamma_G(W)$, where $W = (w_1, w_2, \ldots, w_n)$ is a weight sequence. Since W is closed under conjugation, W, as a set, equals to the disjoint union of elements in some conjugate classes. No loss of generality, assume W is the disjoint union of elements in $C_1, C_2, \ldots, C_m$. Define a ramification χ by χ_C = multiplicity of C in $\{C_1, C_2, \ldots, C_m\}$. Then, we can get that $\Gamma_G(W) \cong \Gamma(G, \chi)$ as direct graphs.

Conversely, let $\Gamma(G, \chi)$ be a Hopf quiver with $\chi = \sum_{C\in\mathcal{C}} \chi_C C$. Define W to be the disjoint union of elements in χ_C copies of C. Since W is a finite set, we can give an order on W such that W is sequence. Clearly, W is a weight sequence, $\Gamma_G(W)$ is a covering quiver and $\Gamma(G, \chi) \cong \Gamma_G(W)$ as direct graphs.

Combining these remarks, we get the following consequence.

Proposition 2.1 *A quiver is a covering quiver if and only if it is a Hopf quiver.* $\square$

The following conclusions (see Theorem 2.3 in [6]) states the importance of covering quivers.

Lemma 2.2 *Let H be a finite dimensional basic Hopf algebra over k. Then there exists a finite group G and a weight sequence $W = (w_1, w_2, \ldots, \ldots, w_n)$ of G, such that $H \cong k\Gamma_G(W)/I$ for an admissible ideal I.* □

Next, let's recall a wonderful E.Green's conclusion, which plays a crucial role in the proof of the main theorem (Theorem 3.1).

There is a natural left G-action on $\Gamma_G(W)$. That is, $g \cdot v_h = v_{hg^{-1}}$ and $g \cdot (a_i, h) = (a_i, gh)$ for $v_h \in \Gamma_G(W)_0$, $(a_i, h) \in \Gamma_G(W)_1$ and $g \in G$. Assume $W = (w_1, w_2, \ldots, \ldots, w_n)$ is the weight sequence. Clearly, the orbit graph, $\Gamma_G(W)/\sim G$, is the graph with one vertex and n loops. Thus, $k\Gamma_G(W)/\sim G$ is isomorphic to the free algebra in n non-commuting variables via assigning to each loop a variable and then assigning each directed path its associated word in the n variables, i.e.

$$k\Gamma_G(W)/\sim G \cong k\{x_1, x_2, \ldots, x_n\}$$

Denote $k\{x_1, x_2, \ldots, x_n\}$ by F. It is a G-graded algebra by giving x_i degree w_i (For detail, see [6]). Since the following conclusion is important, we call it Green Theorem (see Corollary 4.5 in [6]). Note that we freely use some terminologies in [6].

Lemma 2.3 ***(Green Theorem)*** *Let $k\Gamma_G(W)$ be a Hopf algebra with Hopf structure given by an allowable kG-bimodule structure. Let I be an admissible Hopf ideal in $k\Gamma_G(W)$ and F be the G-graded free algebra isomorphic to $k\Gamma_G(W)/\sim G$, which we view as an identification. Finally let $\overline{I}$ be the ideal generated in F by the orbit classes of elements of I. Then*

(a) *$\overline{I}$ is a homogenous ideal in the free algebra F and hence $F/\overline{I}$ is a finite dimensional G-graded algebra.*

(b) *The category of G-graded $F/\overline{I}$-modules (respectively, finite dimensional G-graded $F/\overline{I}$-modules) is equivalent to the category of $k\Gamma_G(W)/I$-modules (resp. finite dimensional $k\Gamma_G(W)/I$-modules).* □

In fact, by the proof of this lemma in [6], when I satisfies the condition $G \cdot I \subset I$, Green Theorem is also true. Since, clearly, $G \cdot J^2 \subset J^2$ (J denote the ideal generated by all arrows), we have the following corollary.

Corollary 2.4 *With notations above. The category of G-graded $F/\overline{J^2}$-modules (respectively, finite dimensional G-graded $F/\overline{J^2}$-modules) is equivalent to the category of $k\Gamma_G(W)/J^2$-modules (resp. finite dimensional $k\Gamma_G(W)/J^2$-modules).* □

3 Main Result

Recall that an algebra is called *Nakayama* if each indecomposable projective left and right module has a unique composition series. The main result of this paper is the following theorem.

Theorem 3.1 *Let H be a finite dimensional basic Hopf algebra over k. Then H is finite representation type if and only if it is Nakayama.*

The sufficiency follows immediately since it is known that every Nakayama algebra is finite representation type ([1], p. 197). In order to prove the necessity, we need some preparations.

Lemma 3.2 *Let G be a group. If $k\{x,y\}$ has a G-graded structure and x,y are homogeneous elements, then $\Lambda = k\{x,y\}/(x,y)^2$ has infinite isoclasses of indecomposable G-graded modules.*

Proof: Clearly, $\Lambda = k\{x,y\}/(x,y)^2 = k[x,y]/(x,y)^2$. Then Λ is a local algebra so that Λ is an indecomposable Λ-module, and is the only indecomposable projective Λ-module up to isomorphism. Let J_Λ denote the Jacobson radical of Λ. Then $J_\Lambda^2 = 0$ and $J_\Lambda \cong S \oplus S$ where $S = \Lambda/J_\Lambda$ is the unique simple Λ-module up to isomorphism.

By assumption, we know x,y are homogeneous elements. Thus, $J_\Lambda = (x)/(x,y)^2 \oplus (y)/(x,y)^2$ is a G-graded submodule of Λ. We claim that every finite dimensional Λ-module is a G-graded module. In fact, for any Λ-module C, there exists a positive integer n such that $n\Lambda$ is the projective cover of C since Λ is the only indecomposable projective Λ-module. That's to say, there is an epimorphism $\pi : n\Lambda \twoheadrightarrow C$ such that $n\Lambda/J_\Lambda n\Lambda \cong C/J_\Lambda C$. Thus, clearly, $Ker(\pi) \subset J_\Lambda n\Lambda$. Since $J_\Lambda^2 = 0$, $J_\Lambda n\Lambda$ is semisimple. More explicitly, it is a direct sum of some copies of $(x)/(x,y)^2$ and $(y)/(x,y)^2$. Therefore, $Ker\pi$ is also a direct sum of some copies of $(x)/(x,y)^2$ and $(y)/(x,y)^2$. This means $Ker\pi$ is a G-graded submodule of $n\Lambda$ and so $C \cong n\Lambda/Ker\pi$ is also G-graded.

By the Example in page 110 in [1], we know Λ has infinite isoclasses indecomposable Λ-modules. Thus, by the claim above, we get the desire conclusion. □

Corollary 3.3 *Let $k\{x_1, x_2, \ldots, x_n\}$ is a G-graded algebra with homogeneous elements $x_1, x_2, \ldots, x_n$. If $n \geq 2$, $k\{x_1, x_2, \ldots, x_n\}/(x_1, x_2, \ldots, x_n)^2$ has infinite isoclasses of indecomposable G-graded modules.*

Proof: Note that there is a natural epimorphisms as G-graded algebras

$$k\{x_1, x_2, \ldots, x_n\}/(x_1, x_2, \ldots, x_n)^2 \overset{\pi}{\twoheadrightarrow} k\{x_1, x_2\}/(x_1, x_2)^2$$

Thus, through the algebra morphism π above, we have every G-graded $k\{x_1, x_2\}/(x_1, x_2)^2$-module is a G-graded $k\{x_1, x_2, \ldots, x_n\}/(x_1, x_2, \ldots, x_n)^2$-module. Therefore, Lemma 3.2

implies this corollary. □

Proof of Theorem 3.1: We only need to prove the necessity now. Let H be a basic Hopf algebra of finite representation type. By Lemma 2.2, there exist a finite group G and a weight sequence $W = (w_1, w_2, \ldots, w_n)$ such that $H \cong k\Gamma_G(W)/I$ for an admissible ideal I. We claim $n \leq 1$. Otherwise, let $n \geq 2$. Recall that $k\Gamma_G(W)/\sim G \cong k\{x_1, x_2, \ldots, x_n\}$, which is a G-graded algebra by giving x_i degree w_i, and $\overline{I'}$ denote the ideal generated in $k\{x_1, x_2, \ldots, x_n\}$ by the orbit classes of elements of I'. Just like before, let J denote the ideal generated by all arrows. Then it is easy to see that $k\{x_1, x_2, \ldots, x_n\}/\overline{J^2} = k\{x_1, x_2, \ldots, x_n\}/(x_1, x_2, \ldots, x_n)^2$. Thus by Corollary 3.3, $k\{x_1, x_2, \ldots, x_n\}/\overline{J^2}$ has infinite isoclasses of G-graded modules. By corollary 2.4, $k\Gamma_G(W)/J^2$ is infinite representation type. Since $I \subset J^2$, there is a natural epimorphism as algebras $k\Gamma_G(W)/I \twoheadrightarrow k\Gamma_G(W)/J^2$. Therefore, $H \cong k\Gamma_G(W)/I$ is infinite representation type too. It is a contradiction. This implies $n \leq 1$.

When $n = 0$, there is no any arrow in $\Gamma_G(W)$. This means H is semisimple and of course Nakayama.

When $n = 1$, $\Gamma_G(W)$ is composed of disjoint union of basic cycles (see Example 2.1). It is well known that an indecomposable elementary algebra is Nakayama if and only if its quiver is a basic cycle or a linear quiver A_m (see [4]). Thus H is Nakayama too. □

Example 3.1 Let q be a n-th primitive root of unity. Recall that the Taft algebra $T_{n^2}(q)$ is an Hopf algebra generated by elements g and x, with relations

$$g^n = 1, \quad x^n = 0, \quad xg = qgx$$

with comultiplication Δ, counit ε, and antipode S given by

$$\Delta(g) = g \otimes g, \quad \Delta(x) = 1 \otimes x + x \otimes g$$

$$\varepsilon(g) = 1, \qquad \varepsilon(x) = 0$$

$$S(g) = g^{-1}, \qquad S(x) = -xg^{-1}$$

It is a basic Hopf algebra (This fact can be gotten from two known results, $T_{n^2}(q) \cong T_{n^2}(q)^*$ and $T_{n^2}(q)$ is a point Hopf algebra). We claim that it is Nakayama (This conclusion can also be deduced from [2]).

Denote $T_{n^2}(q)$ by A. Then $J_A = span\{g^i x^j | i = 0, 1, \ldots, n-1,\ j = 1, 2, \ldots, n-1\}$, the linear span of $\{g^i x^j\}_{0 \leq i \leq n-1,\, 1 \leq j \leq n-1}$ and thus $J_A^2 = span\{g^i x^j | i = 0, 1, \ldots, n-1,\ j = 2, 3, \ldots, n-1\}$. Denote A/J_A^2 by Λ and the socle of A/J_A^2 by $Soc\Lambda$. Then it is easy to see that $Soc\Lambda = J_A/J_A^2 = span\{g^j x + J_A^2 | j = 0, 1, \ldots, n-1\}$. Define a linear isomorphism

$$f:\ \Lambda/J_\Lambda \rightarrow Soc\Lambda \ \text{ by } \ (g^i + J_A^2) + J_\Lambda \mapsto g^i x + J_A^2$$

for $i = 0, 1, \ldots, n-1$. Clearly, f is also a Λ-module map. Thus $\Lambda/J_\Lambda \cong Soc\Lambda$ as Λ-modules which implies Λ is a self-injective algebra (see exercise 12 in p. 135 in [1]). Therefore Λ is Nakayama by Proposition 2.16 in p.119 in [1]. It is known an algebra B is Nakayama if and only if B/J_B^2 is Nakayama. Since $\Lambda = A/J_A^2$, $A = T_{n^2}(q)$ is Nakayama. So it is finite representation type.

Remark 3.4 *In the view point of representation theory, Nakayama algebras is the best understood artin algebras next to semisimple algebras. Many properties of its representation are known. For example, we can draw its Auslander-Reiten quivers directly(see [1]). When a Nakayama algebra is elementary, its Gabriel quiver is either a basic cycle or a linear quiver A_m. Thus the Gabriel and Auslander-Reiten quivers of basic Hopf algebras of finite representation type are clear.*

4 Classification

In this section we will classify all finite dimensional basic Hopf algebras of finite representation type. When H is semisimple, we have $H = H/J_H \cong k \times k \times \cdots \times k$. That's to say H is commutative. A classical result states that H is a finite dimensional commutative Hopf algebra over a algebraically closed field k if and only if $H \cong (kG)^*$ for a finite group. So, our main task is to classify them in non-semisimple case.

Recall an algebra is called *monomial* if there exists a quiver Γ and an admissible ideal I generated by some paths such that $A \cong k\Gamma/I$. A coalgerba C is called *comonomial* if C^* is a monomial algerba. A finite dimensional Hopf algebra is called *monomial* (resp. *comonomial*) Hopf algebra if it is monomial (resp. comonomial) as algebra (resp. coalgebra). So, it is obviously that a finite dimensional Hopf algebra H is a monomial Hopf algebra if and only if H^* is a comonomial Hopf algebra. One of key observations we need in our study is the following lemma which was proved in [2] (see Corollary 2.4 in [2]).

Lemma 4.1 *A non-semisimple Hopf algebra is a monomial Hopf algebra if and only if it is elementary and Nakayama.* □

Therefore, combining Theorem 3.1, we have the following corollary.

Corollary 4.2 *Let H be a non-semisimple Hopf algebra over a algebraically closed field k. Then H is a basic Hopf algebra of finite representation type if and only if it is a monomial Hopf algebra.* □

So, in order to classify basic Hopf algebras of finite representation type, it is sufficient to classify monomial Hopf algebras. In [2], the authors classified all comonomial Hopf algebras when characteristic of k is 0 (see Theorem 5.9 in [2]). Let us recall it.

Lemma 4.3 *Let k be a algebraically closed field with characteristic 0. There is a one to one correspondence between sets*

$$\{\text{the isocalsses of non-semisimple comonomial Hopf } k\text{-algebras}\}$$

and

$$\{\text{the isoclasses of group data over } k\} \quad \square$$

In the above lemma, a group data (for detail, see [2]) over k is defined to be sequence $\alpha = (G, g, \chi, \mu)$ consists of

(1) a finite group G, with an element g in its center,

(2) a one-dimensional k-representation χ of G,

(3) an element $\mu \in k$ such that $\mu = 0$ if $o(g) = o(\chi(g))$, and if $\mu \neq 0$ then $\chi^{o(\chi(g))} = 1$.

Remark 4.4 *For a group datum $\alpha = (G, g, \chi, \mu)$ over k, the corresponding comonomial Hopf algebra $A(\alpha)$ was defined in [2], which is generated as an algebra by x and all $h \in G$ with relations*

$$x^d = \mu(1 - g^d), \quad xh = \chi(h)hx, \quad \forall\, h \in G$$

where $d = o(\chi(g))$. Its comultiplication Δ, counit ε, and antipode S defined by

$$\Delta(x) = g \otimes x + x \otimes 1, \quad \varepsilon(x) = 0,$$

$$\Delta(h) = h \otimes h, \quad \varepsilon(h) = 1 \quad \forall\, h \in G,$$

$$S(x) = -g^{-1}x, \quad S(h) = h^{-1}, \quad \forall\, h \in G.$$

For any quiver Γ, we define $C_d(\Gamma) := \oplus_{i=1}^{d-1} k\Gamma(i)$ for $d \geq 2$, where $\Gamma(i)$ is the set of all paths of length i in Γ. We denote the basic cycle of length n (Example 2.1) by Z_n and denote $C_d(Z_n)$ by $C_d(n)$. In [7], we get the following conclusion (see Theorem 4.2 in [7]).

Lemma 4.5 *Let H be a non-semisimple comonomial Hopf algebra over k of characteristic p. Then there exist a d_0-th primitive root of unity $q \in k$ with $d_0|n$, $r \geq 0$ and $d = p^r d_0 \geq 2$ such that*

$$H \cong C_d(n) \oplus \cdots \oplus C_d(n)$$

as coalgebras and

$$H \cong C_d(n) \#_\sigma k(G/N)$$

as Hopf algebras, where $G = G(H)$, the set of group-like elements of H, and $N = G(C_d(n))$, the set of group-like elements of $C_d(n)$. $\square$

Summarizing above conclusions, we have the following classification theorem of finite-dimensional basic Hopf algebras of finite representation type.

Theorem 4.6 (A) *Let H be a finite-dimensional basic Hopf algebra of finite representation type. Then*

(A.1) *If H is semisimple, then $H \cong (kG)^*$ for some finite group G;*

(A.2) *If H is not semisimple and the characteristic of k is zero, then $H^* \cong A(\alpha)$ for some group datum $\alpha = (G, g, \chi, \mu)$ where $A(\alpha)$ was defined in above Remark;*

(A.3) *If H is not semisimple and the characteristic of k is p, then there exist a d_0-th primitive root of unity $q \in k$ with $d_0|n$, $r \geq 0$ and $d = p^r d_0 \geq 2$ such that*

$$H^* \cong C_d(n) \oplus \cdots \oplus C_d(n)$$

as coalgebras and

$$H^* \cong C_d(n) \#_\sigma k(G/N)$$

as Hopf algebras, where $G = G(H)$ and $N = G(C_d(n))$.

(B) *Let H be a finite dimensional Hopf algebra. If*

(B.1) *$H \cong (kG)^*$ for some finite group G or*

(B.2) *$H^* \cong A(\alpha)$ for some group datum $\alpha = (G, g, \chi, \mu)$ where $A(\alpha)$ was defined in above Remark or*

(B.3) *$H^* \cong C_d(n) \oplus \cdots \oplus C_d(n)$ as coalgebras,*
then H is a basic Hopf algebra of finite representation type.

Proof: (A.1) is explained in the first paragraph of this section. By using Lemma 4.3 and Lemma 4.5, (A.2) and (A.3) can be gotten directly as long as we note that H^* is a comonomial Hopf algebra now.

Since $(kG)^*$ is semisimple and clearly basic, (B.1) implies H is a basic Hopf algebra of finite representation type.

By Lemma 4.3 and Remark 4.4, $A(\alpha)$ is a comonomial Hopf algerba and thus (B.2) implies H is a monomial Hopf algebra. Therefore, by Corollary 4.2, H is a basic Hopf algebra of finite representation type.

It is known that $C_d(n)$ is a comonomial coalgebra (see [2]). From this fact we know that (B.3) implies H^* is a comonomial Hopf algebra and thus H is a monomial Hopf algebra. That is, H is a basic Hopf algebra of finite representation type. □

Remark 4.7 (1) *In order to not cause confusion, we introduced the concept of comonomial Hopf algebras. Note that, in [2] and [7], comonomial Hopf algebra in this paper was called monomial Hopf algebra.*

(2) *Part (A) of Theorem 4.6 gives the structures of finite dimensional basic Hopf algebras of finite representation type. In certain sense, Part (B) of Theorem 4.6 is the converse of Part (A) since Part (B) implies that the structures given in Part (A) are precisely all finite dimensional basic Hopf algebras of finite representation type.*

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