Lecture I

Lectures on D-branes
& nonperturbative formulations of string theory

Hangzhou, China
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0. Intro (1/2 lecture)
1. D-branes (2 lectures) — basic tool in string/M thy.
   — rich mathematical structure
2. Matrix model of M-theory (1.5-2 l) — q. thy of gravity in $\mathbb{R}^{10,1}$
   — math. structure needs generalization
3. String field theory (1.5-2 l) — "background independent" string thy.
   — math. structure not understood.

0. Intro [brief history of S.T. to date]

Local forces: EM, wk. nuclear, Str. nuclear } QFT

gravity } classical GR

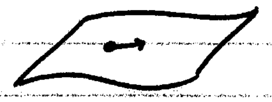
Global structure: cosmology (initial conditions, topology of space, ...)

Physics goal:
Math. description of (all of) nature

Need quantum gravity.

Relevant
- near black holes
- in early universe

Quantum Mechanics



classical $x_t \in M$ $\Rightarrow$ quantum $\psi_t \in \mathcal{H} = \mathcal{L}^2(M)$

symmetry group G $\longrightarrow$ finite dim. irreps in $\mathcal{H}$

$\rightarrow$ quantization of L, E, etc..

QFT

field $\phi(x)$, $A_\mu(x)$ $\Rightarrow$ functional $\Psi[\phi(x)]$

Poincaré gp $\longrightarrow$ quanta = particles

Gravity $\not\Rightarrow$ QFT

Strings:

quanta

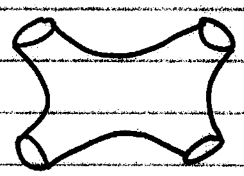
closed strings  $\sigma_t : S^1 \rightarrow M$

open "  $\sigma_t : [0, \pi] \longrightarrow M$

$\sigma_t(0), \sigma_t(\pi) \in \mathcal{D} \subset M$

"D-branes"

Perturbative string theory [quantize strings on w-sheet, PI formalism]



$\longrightarrow$ space-time M
(flat: $M = \mathbb{R}^{9,1}$)

world-sheet Σ
(Riemann surface)

$$X : \Sigma \to M \quad \Rightarrow \quad X^\mu(\sigma, \tau) \text{ locally}$$

compute $Z = \int \mathcal{D}X \, e^{-S[X]}$, + correlation functions

[Ham. Dav]

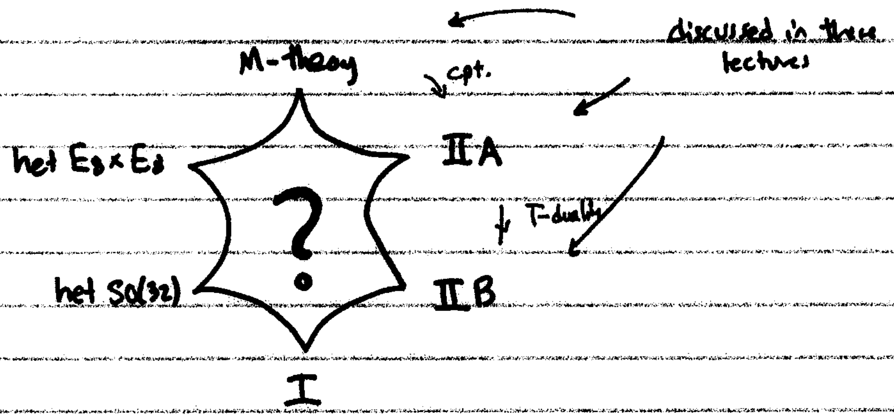
excited closed strings $\Longrightarrow$ massless particles in Poincaré spin 2 rep.

$\Longrightarrow$ String thy = quantum gravity
(perturbatively)

Circa 1995:

- 5 superstring theories known in 10D:
  IIA, IIB, I, heterotic $E_8 \times E_8$, $SO(32)$
- only known perturbatively ($Z = \sum_{\text{genus } h} Z_h \, g^h$)
- 4D physics from $M^{10} = \mathbb{R}^{3,1} \times CY^6$
  [Yau, de Alwis]

Now:



Dualities:

- Relate degrees of freedom between pert. theories
- some dualities nonperturbative $(g \to 1/g)$
- Known theories = different perturbative limits of 1 (unknown) theory

M-theory:

- Quantum SUGRA in 11D
- No micro description known [M(embranes)? M(atrices)?]
- Related to IIA through

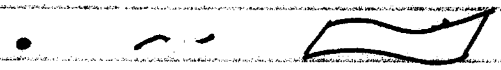
$$\text{IIA on } M^{10} \Longleftrightarrow M \text{ on } M^{10} \times S^1$$

$$g_{IIA} \cdot l_s \Longleftrightarrow R_{11} = \text{radius of } S^1$$

$$\left(g \to \infty, R^{9,1} \Longrightarrow M : \mathbb{R}^{10,1} \right)$$

- Het $E_8 \times E_8$ on M^{10} = M on $M^{10} \times S^1/\mathbb{Z}_2$

Branes: D-branes / M-branes : higher-dimensional extended objects



D∅ D1 D2, M2

$$\left.\begin{array}{ll} M: & M2,\ M5 \\ IIA: & D\emptyset,\ D2,\ D4,\ldots \\ IIB: & D1,\ D3,\ D5 \end{array}\right\}\text{ preserve } 1/2 \text{ SUSY}$$

brane democracy/equality:
  Strings & branes equally fundamental
   – related through dualities

Branes ⟶ new perspectives/results (part 1 of lectures)

- ⇒ QFT (QCD & SUSY relatives)
- ⇒ Mathematics (gauge theories)
- ⇒ Black holes (information puzzle)
- ⇒ cosmology ("brane world")
- ⇒ nonperturbative string theory

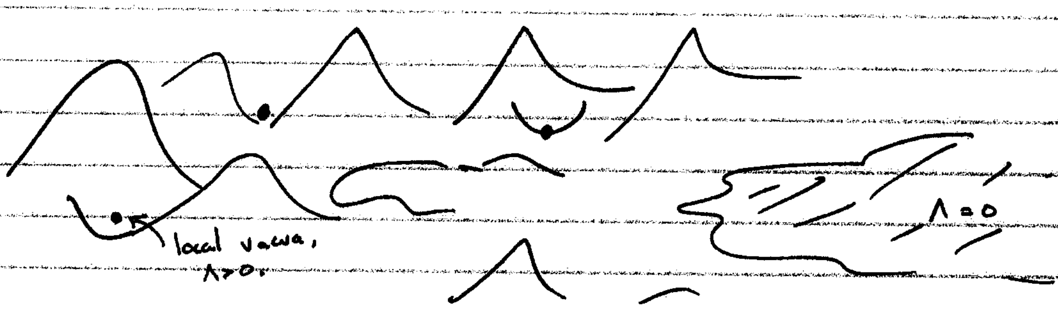
Nonperturbative approaches to string theory

- M(atrix theory):
  M-theory on $\mathbb{R}^{10,1}$ = Matrix quantum mechanics
  (part 2 of lectures)

- AdS/CFT:
  String theory on $AdS_5 \times S^5$ = $\mathcal{N}=4$ Super YM on $\mathbb{R}^{3,1}$, $G = \lim_{N\to\infty} U(N)$
  (Cai, Gopakumar)

Matrix thy, AdS/CFT valid in fixed asymptotic space-time.

For cosmology, need background-independent formulation

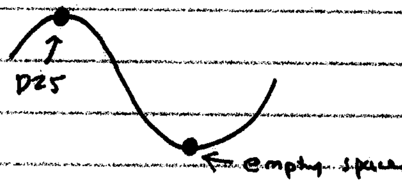


"string landscape"

Only known background-independent formulation:

String field theory

→ recent progress after Sen conjectures



SFT → part of "open string landscape"

(part 3 of lectures)

Lecture II

1. D-branes

Ref: Polchinski: hep-th/9611050
wT: " /9801182

1.1 D-branes & RR charges

2 descriptions of D-branes:

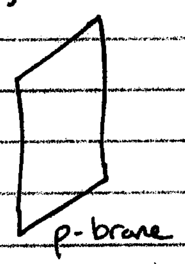
a) Supergravity:

10D SUGRA theories have p-form fields in graviton multiplet

Ex. IIA: $A^{(1)}_{\mu}$, $A^{(3)}_{\mu\nu\lambda}$

IIB: $A^{(0)}$, $A^{(2)}_{\mu\nu}$, $A^{(4)}_{\mu\nu\lambda\rho}$

$\exists$ charged brane-like SUGRA solutions (~charged BH's)



Charged under $A^{(p+1)}$

(coupling $\int_\Sigma A^{(p+1)}$

$\sim \int e A_\mu dx^\mu$ for electron)

preserve 1/2 of SUSY

b) String theory:

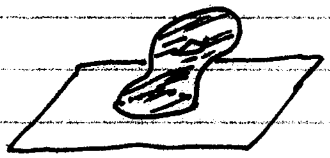
locus for open string boundary

Dp-brane: $X^\mu(0,\pi) = 0$, $\mu \in \{p+1, \dots, 9\}$

Polchinski (1995): D-branes carry RR charges.

Identify with RR-charged SUGRA p-branes

1.2 BRST quantization of open strings (bosonic)



$X : \Sigma \to \mathbb{R}^{9,1}$

$$S_{NG} = -\frac{1}{4\pi\alpha'}\int \sqrt{-\det(\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu})}$$

Σ R. surface w/ bdry

γ_{ab} auxiliary world-sheet metric

$$\Rightarrow \quad S_P = -\frac{1}{4\pi\alpha'}\int \sqrt{-\gamma}\,\gamma^{ab}\partial_a X^\mu \partial_b X_\mu$$

when use eom for γ.

– gauge fix $\gamma_{ab} = \delta_{ab}$ [conformal gauge, use diff. symmetry]

– BRST gauge-fixing $\Rightarrow$ ghost c, antighost b [GSW, Polchinski]

$$\Rightarrow \quad S = -\frac{1}{4\pi\alpha'}\int \partial_a X^\mu \partial^a X_\mu + \frac{1}{\pi}\int (b_{++}\partial_- c^+ + b_{--}\partial_+ c^-)$$

free bosons

<u>Mode expansion</u>

$$X^\mu(\sigma,\tau) = x_0^\mu + p^\mu \tau + \sum_{n\neq 0}\frac{i}{n}\alpha_n^\mu \cos(n\sigma)e^{-in\tau}$$

[$l=1$, $\alpha' = 1/2$]
[NN BC's]

QM : $X \longrightarrow$ operator

$$[x_0^\mu, p^\nu] = i\eta^{\mu\nu}$$

$$[\alpha_m^\mu, \alpha_n^\nu] = m\eta^{\mu\nu}\delta_{m+n,0}$$

$$(\alpha_n^\mu)^\dagger = \alpha_{-n}^\mu$$

[reality cond.]

Ghosts:

$$c^{\pm}(\sigma,\tau) = \sum_{m=-\infty}^{\infty} c_n e^{\mp in(\sigma \pm \tau)} \qquad \text{(ghost \# 1)}$$

$$b_{\pm\pm}(\sigma,\tau) = \sum_{m=-\infty}^{\infty} b_n e^{\mp in(\sigma \pm \tau)} \qquad (\text{" " } -1)$$

$$\{c_n, b_m\} = \delta_{n+m,0}$$

$$\{c_n, c_m\} = \{b_n, b_m\} = 0$$

Fock space

vacuum $|0;k\rangle$: (gh # 1)

$$b_n |0;k\rangle = 0 \qquad n \geq -1$$
$$c_n |0;k\rangle = 0 \qquad n \geq 2$$
$$\alpha^\mu_n |0;k\rangle = 0, \qquad n \geq 1$$
$$p^\mu |0;k\rangle = k^\mu |0;k\rangle$$

General state:

$$\prod \alpha^{\mu_1}_{-n_1} \alpha^{\mu_2}_{-n_2} \cdots \alpha^{\mu_i}_{-n_i} c_{-m_1} \cdots c_{-m_j} b_{-\ell_1} \cdots b_{-\ell_p} |0;k\rangle$$
$$n_q \geq 1, \; m_q \geq -1, \; \ell_q \geq 2$$

these are states in <u>single string Fock space</u>

(not a Hilbert space; $\eta^{00} = -1$)

BRST operator Q_B

$$Q_B = \sum_{n=-\infty}^{\infty} c_n L^{(m)}_{-n} + \sum_{n,m=-\infty}^{\infty} \frac{(m-n)}{2} :c_m c_n b_{-m-n}: - c_0$$

$$L^{(m)}_q = \begin{cases} \frac{1}{2} \sum_n \alpha^\mu_{q-n} \alpha_{\mu\, n} & q \neq 0 \\ \frac{1}{2} p^2 + \sum_{n=1}^{\infty} \alpha^\mu_{-n} \alpha_{\mu n} & q = 0 \end{cases}$$

Properties of Q_B

- $Q_B^2 = 0$ (in $D = 26$) [nilpotent]
- $\{Q_B, b_0\} = L_0^{(m)} + L_0^{(g)}$
- Q_B has ghost # = 1
- Physical states = cohomology of Q_B (@ ghost # = 1)

$$\mathcal{H}_{phys} = \mathcal{H}_{closed} / \mathcal{H}_{exact}$$
$$= \{|\psi\rangle : Q|\psi\rangle = 0\} \Big/ |\psi\rangle \sim |\psi\rangle + Q_B|\chi\rangle$$

- can choose physical states: $b_0|\psi\rangle_{phys} = 0$.

Convenient to write: $Q = c_0 L_0 + b_0 M + \tilde{Q}$

$$L_0 = \underbrace{\sum_{n=1}^{\infty}\left(\alpha_{-n}\alpha_n + n c_{-n} b_n + n b_{-n} c_n\right)}_{\text{level } N} - 1 + \alpha' p^2$$

Examples of physical states

$|0;p\rangle$ physical iff $p^2 = 1/\alpha' = -M^2$

(Tachyon)

$\varepsilon_\mu \alpha^\mu_{-1} |0;p\rangle$ physical when $p^2 = M^2 = 0$

→ also need $p \cdot \varepsilon = 0$ from $\tilde{Q} \sim c_{-1}\, p \cdot \alpha$.

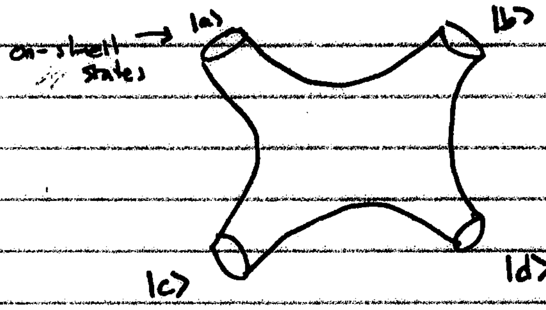
(Gauge field)

States of string $\Rightarrow$ fields in space-time

$|0;p\rangle \Rightarrow \phi(p)$, etc..

Use in 2 ways

(i) perturbative calcs



(ii) Off-shell SFT action

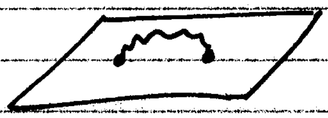
$$\mathcal{A} = \int d^{26}p \left[\phi(p)\, |0;p\rangle + A_\mu \alpha^\mu_{-1} |0;p\rangle + \dots \right]$$

[part 3]

1.3 D-brane world-volume theory

Quantize superstring on $\mathbb{R}^{p,1} \subset \mathbb{R}^{9,1}$

SUSY : $X^\mu \longleftrightarrow$ fermions ψ^μ



Quantize $\Rightarrow$ α^μ_n, ψ^μ_r as above

Define $(-1)^F |0\rangle = -|0\rangle$, $(-1)^F \psi^\mu_r (-1)^F = -\psi^\mu_r$

Consistent subsector : $(-1)^F |s\rangle = |s\rangle$

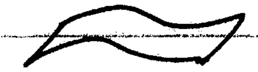
$\Rightarrow$ no tachyon.

Massless fields on $\mathbb{R}^{p,1}$

$\alpha^{\alpha}_{-1}|0\rangle \Rightarrow A_\alpha \quad : \quad \alpha = 0, 1, \ldots, p \qquad$ 1-form

$\alpha^{a}_{-1}|0\rangle \Rightarrow X^a \quad : \quad a = p+1, \ldots, 9 \qquad 9-p$ scalars $\Rightarrow$ describe transverse motion

+ fermions



general background : $\begin{cases} X^\mu : \Sigma_{p+1} \Rightarrow M^{10} \\ A_\alpha \ \text{U(1) gauge field on } \Sigma_{p+1} \\ \Phi, g_{\mu\nu}, B_{\mu\nu} \ \text{on } M^{10} \end{cases}$

Φ dilaton, $g_{\mu\nu}$ metric, $B_{\mu\nu}$ antisymm. (from $\alpha^\mu_{-1}\tilde{\alpha}^\nu_{-1}|0\rangle$ in closed string)

conformal symmetry $\Downarrow$

$$\boxed{S_{BI} = -T_p \int d^{p+1}\xi \, e^{-\Phi} \sqrt{-\det(G_{\alpha\beta} + B_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})} + \text{fermions} + \text{Chern-Simons} + \text{corrections}}$$

Φ, B, G pulled back from M.

Φ = dilaton $\quad e^{-\Phi} = \frac{1}{g}$

$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$

$\tau_p = \frac{1}{g} T_p = \frac{1}{g\, l_s (2\pi l_s)^p} \qquad l_s = \sqrt{\alpha'}$

Low-energy limit : $\quad S_{BI} \rightarrow \underbrace{\tau_p(\text{Volume})}_{\gg} + \underbrace{S_{YM} + \cdots}_{\rightarrow 0}$

Assume
- $g_{\mu\nu} = \eta_{\mu\nu}$
- $B_{\mu\nu} = 0$
- $A^{(p)}_{RR} = 0 \qquad$ [not known how to define if $A^{(p)}_{RR} \neq 0$]
- static gauge $X^\alpha = \xi^\alpha$
- $\partial_\alpha X^a \approx 2\pi\alpha' F_{\alpha\beta} \ll 1$

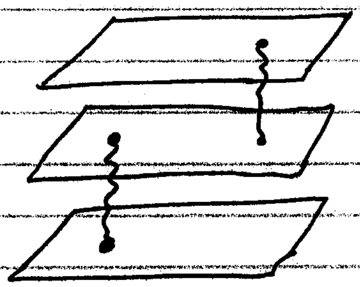
$$\Rightarrow \quad S_{YM} = \tau_p \int \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\psi}\Gamma^\mu \partial_\mu \psi \right)$$

$F_{\alpha\beta}$ = U(1) curvature $\qquad F_{\alpha a} = \partial_\alpha X^a$.

Lecture III

1.4 Multiple D-branes

Consider N $\parallel$ D-branes



$A_\alpha, X^a \Rightarrow N \times N$ matrices
("Chan-Paton" factors)

$S_{BI} \Rightarrow$ "nonabelian Born-Infeld"

$$S \sim \int \mathrm{Tr} \sqrt{-\det\left(G + B + 2\pi\alpha' F\right)}$$

Not yet well-defined: ordering ambiguities

Low-E limit

$$S_{YM} = \tau_p \int \mathrm{Tr}\left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi\right)$$

$$\Downarrow$$

$$F_{ab} = -i\left[X^a, X^b\right]$$

$$D_b \psi = -i\left[X^b, \psi\right]$$

(p+1) dimensional reduction of
U(N) $\mathcal{N}=4$ $D=10$ Super YM.
A_μ = gauge field
ψ = 16-cpt spinor of SO(9,1)

So: at low energies,

$\boxed{\text{N parallel D-branes are described by Yang-Mills theory}}$

1.5 T-duality on Dp-branes [from YM]

T-duality:

$$\mathrm{IIA}/S^1_{(R)} \longleftrightarrow \mathrm{IIB}/S^1_{(\hat{R})}$$

$$R \longleftrightarrow \hat{R} = \alpha'/R$$

Closed strings:

winding  n

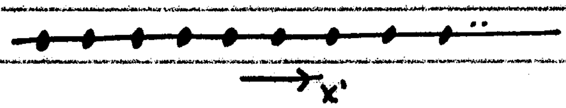
momentum  m

$$E_{n,m} = \frac{1}{2\pi\alpha'}(2\pi n R) + \frac{m}{R} = \frac{nR}{\alpha'} + \frac{m}{R}$$

$$E_{n,m}(R) = E_{m,n}(\alpha'/R)$$

Open strings:

Consider $D0$ in IIA on $S^1_{(R)}$. $[2\pi\alpha' = 1]$

 $\Rightarrow$ covering space 

$X^a \Rightarrow N = \infty \times N = \infty$ matrices (operators)

Background

$$\tilde{X}^1 = \begin{pmatrix} -4\pi R & & & & \\ & -2\pi R & & & \\ & & 0 & & \\ & & & 2\pi R & \\ & & & & 4\pi R \end{pmatrix} \qquad \begin{aligned} \tilde{A}_0 &= 0 \\ \tilde{X}^{k\neq 1} &= b^k \mathbb{1} \end{aligned}$$

Define $U = \begin{pmatrix} 0 & 1 & 0 & & & \\ 0 & 0 & 1 & 0 & \cdots & \\ & 0 & 0 & 1 & 0 & \\ \cdot^{\cdot^{\cdot}} & & 0 & 0 & 1 & \\ & & & 0 & 0 & \ddots \end{pmatrix}$

$$X^a = \tilde{X}^a + \delta X^a$$

Fluctuations satisfy
$$U X^{k\neq 1} U^{-1} = X^k$$
$$U X^1 U^{-1} = X^1 + 2\pi R$$
$$U A_0 U^{-1} = A_0$$

$$X^1 = \begin{pmatrix} \cdots & 0+f_0 & f_1 & f_2 & \cdots \\ \cdots & f_{-1} & 2\pi R + f_0 & f_1 & f_2 \cdots \end{pmatrix}$$

Matrix rep of $D^1 = i\partial_1 + A_1(\hat{x})$

on $\phi(\hat{x}) = \sum_n \hat{\phi}_n e^{in\hat{x}/\hat{R}} \Rightarrow \begin{pmatrix} \vdots \\ \hat{\phi}_1 \\ \hat{\phi}_0 \\ \hat{\phi}_{-1} \\ \vdots \end{pmatrix}$

$\hat{R} = \alpha'/R = 1/2\pi R$; $\quad A_1(\hat{x}) = \sum_n \hat{a}_n e^{in\hat{x}/\hat{R}}$

so $X^1 \to D^1$, $\quad X^k \to X^k(\hat{x})$, $A_0 \to A_0(\hat{x})$

$$S_{YM}^{(0+1)} = \frac{1}{2}\int d\tau \left[(D_0 X^a)^2 - \sum_{a<b} [X^a, X^b]^2\right]$$

$$\Rightarrow \frac{1}{2}\int d\tau\, d\hat{x} \left[(D_0 X^k)^2 - F_{01}^2 + (D_1 X^k)^2 - \sum_{k<l} [X^k, X^l]^2\right]$$

$$= S_{YM}^{(1+1)}$$

Same story w/ fermions

so FT on D0 on $S^1(R)$ = FT on D1 on $S^1(\hat{R})$

T-duality in SYM

Note: including units, $X \to (2\pi\alpha')(i\partial + A)$

Note: can generalize to

$$U X^k U^{-1} = \Omega (X^k + \delta^{k1} 2\pi R_1 \mathbb{1}) \Omega^{-1}$$

Ω: gauge xform b.c. $A(\hat{x} + 2\pi\hat{R}) = \Omega A(\hat{x}) \Omega^{-1}$

On T^d can generate any $U(N)$ bundle topology

If U_i, U_j don't commute $\Rightarrow$ Noncommutative YM theory (Douglas)

Note: in general,

D_p ~~unwrapped~~ wrapped on S^1_R $\longleftrightarrow$ $D_{p\mp 1}$ ~~wrapped~~ unwrapped on $S^1_{\hat{R}}$.

1.6 Branes from branes (Application of T-duality)

Additional terms in N Dp-brane action

$$S_{CS} = \int_{\Sigma_{p+1}} \operatorname{Tr} A\, e^{\frac{1}{2\pi}(F+B)} \hat{A}(\mathcal{R})$$

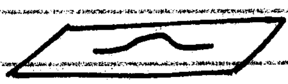
Green, Harvey, Moore 9605033
Freed & Witten (K-th) 9907189
Ellwood 04mmnnn

$A = \sum_k A^{(k)}$ sum over RR fields

$\hat{A}(\mathcal{R}) = \hat{A}$ genus, $\to \sqrt{\hat{A}(T\mathcal{R}) / \hat{A}(N\mathcal{R})}$ in curved space

Curvature sources RR fields

Ex. $\frac{1}{2\pi}\int_{\Sigma_{p+1}} A^{(p-1)} \wedge F$



$\Rightarrow F_{\alpha\beta}$ on p-brane carries (p-2)-brane charge

On N compact Dp-branes

$\frac{1}{2\pi}\int \mathrm{Tr}\, F$: (p-2)-brane charge

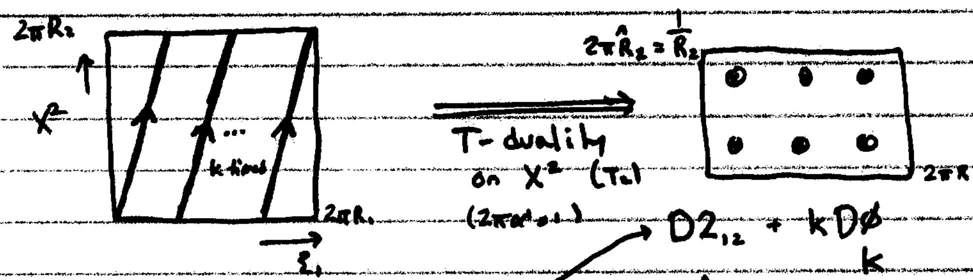
$\frac{1}{8\pi^2}\int \mathrm{Tr}\, F \wedge F$: (p-4)-brane charge

⋮

Also couplings from curvature - e.g. Dp on K3 carries Dp-4 charge

<u>Example from T-duality</u>

Diagonal D1 on T^2



$D1_1 + k\, D1_2 \rightarrow D2_{12} + k\, D0$

$X^2 = k\, R_2/R_1\, \xi_1 \rightarrow A_2 = \frac{k}{2\pi \hat{R}_2 R_1}\xi_1$

$F_{12} = \frac{k}{2\pi \hat{R}_2 R_1}$

$\frac{1}{2\pi}\int F_{12} = k = D0$ charge.

(in YM thy on $D1_1$ coord. ξ_1)

T-duality on X^1:

Note: $X(\xi + 2\pi R_1) = U X(\xi) U^{-1}$ @ bdry.

($\mathbb{Z}$ indices $\leftrightarrow$ images in X^2)

$$D1_1 + k\, D1_2 \xrightarrow{T_1} D0 + k\, D2_{12}$$

$$D' \longrightarrow X'$$

$$D_1 X^2 = k\, R_2/R_1 \longrightarrow [X^1, X^2] = 2\pi i\, R_2 \hat{R}_1 k\, \mathbb{1}$$

$\Rightarrow$ Dp-branes from Dp-2 branes:

$2\pi i\,[X^a, X^b]$ on Dp-brane has Dp+2-brane charge

(e.g. here, k D2-branes wrapped on $4\pi^2 R_2 \hat{R}_1$ area T^2)

Higher brane charges: Interpretation

1) $\mathrm{Tr}\,[X, Y] \neq 0$ when $N = \infty$,
$\Rightarrow$ wrapped branes on compact spaces.

2) Even if $\mathrm{Tr}[X, Y] = 0$, N finite,
can have multipole moments.

[Came from M(atrix) thy. (next!)
Taylor, Van Raamsdonk 9812239, 9910052
Myers 9910053]

Ex. matrix sphere on N D0-branes

$$X^a = \frac{2r}{N} J^a, \quad a = 1,2,3 \qquad J^a \text{ N-dim. generators of } SU(2).$$

"matrix sphere" 

- $X^{1^2} + X^{2^2} + X^{3^2} = r^2 \mathbb{1} + O(1/N^2)$
- rotationally invariant (up to gauge xform)

$\mathrm{Tr}\,[X^a, X^b] = 0$, but $\mathrm{Tr}\,[X^1, X^2] X^3 \neq 0 \Rightarrow$ D2-brane dipole moment

Lectures IV, V, VI

2. Matrix theory

2.1 Introduction

YM action for N D0-branes (matrix QM)

$$S = \int d\tau \, \mathrm{Tr}\left(\tfrac{1}{2}(D_\tau X^i)^2 - \tfrac{1}{4}[X^i, X^j][X^i, X^j] + \tfrac{1}{2}\theta^T \gamma_i [X^i, \theta] \right)$$

$X^{i=1,\dots,9}$ N×N matrices

θ 16-component SO(9) spinor

Claim: as $N \to \infty$,

(matrix theory conjecture) S describes quantum gravity in flat 11D spacetime (M-theory)!

2 approaches:

i) Quantize (super) M2 in 11D (Goldstone, de Wit, Hoppe, Nicolai 1988)

ii) Consider $N \to \infty$ D0's in IIA (Banks, Fischler, Shenker, Susskind, 1997)

Evidence for conjecture:

- "Objects" of M-theory correctly reproduced
  - gravitons
  - membranes
  - 5-branes
- classical linear gravity from 1-loop effects in MQM
- some nonlinear terms correctly reproduced by 2-loop etc. calcs.

2.2 M-theory $\longleftrightarrow$ IIA

Compactification $M^{11} = M^{10} \times S^1$

antisymmetric 3-form field

11D SUGRA $\qquad G_{IJ}, \quad C_{IJK}, \quad$ ferms $\quad I, \ldots \in \{0, 1, \ldots, 9, 11\}$

$\Downarrow S_1 \qquad\qquad \Downarrow \qquad\qquad \Downarrow$

10D SUGRA

$$g_{\mu\nu} = G_{\mu\nu} \qquad A^{(3)}_{\mu\nu\lambda} = C_{\mu\nu\lambda}$$
$$A^{(1)}_{\mu} = G_{\mu 11} \qquad B_{\mu\nu} = C_{\mu\nu 11}$$
$$\phi = G_{11\,11}$$

KK modes

momentum modes on $S^1 \implies$ KK modes in 10D

e.g. graviton w/ $p = N/R \implies$ bound state of N D$\emptyset$'s.

Branes

M-theory has M2, M5-branes.

"unwrapped" M2 $\implies$ D2

"wrapped" M2 $\implies$ F1

"unwr." M5 $\implies$ NS 5-brane (magnetic source for $B_{\mu\nu}$)

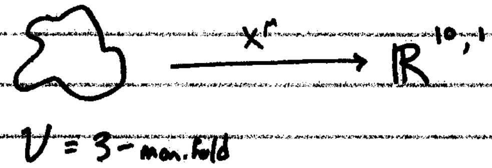
"wr." M5 $\implies$ D4.

Radius R of S^1 : $\quad R \longrightarrow g\, l_s$ in IIA.

"Defines" M-theory on $\mathbb{R}^{10,1}$ as $g \to \infty$ limit of IIA on $\mathbb{R}^{9,1}$.

2.3 Classical membrane

Begin like with string



$$S_{NG} = -T\int d^3\sigma \sqrt{-\det h_{\alpha\beta}}$$

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

~~add world volume metric~~

$$\Rightarrow \quad S_P = -\frac{T}{2}\int d^3\sigma \sqrt{-\gamma}\left(\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu - 1\right)$$

(solve EOM for $\gamma \Rightarrow S_{NG}$) [needed due to lack of scale invariance]

Gauge fix: $\gamma_{\alpha\beta} \rightarrow 6$ DOF
only 3 diff. symmetries

Fix $\gamma_{0a} = 0$
$\gamma_{00} = -\frac{4}{\nu^2}\bar{h} \equiv -\frac{4}{\nu^2}\det h_{ab}$ $\quad a,b \in \{1,2\}$
(ν const.)

$$S \rightarrow \frac{T\nu}{4}\int d^3\sigma\left(\dot{X}^\mu \dot{X}_\mu - \frac{4}{\nu^2}\bar{h}\right).$$

Introduce

$$\{f,g\} = \varepsilon^{ab}\partial_a f\,\partial_b g, \qquad \varepsilon^{12} = -\varepsilon^{21} = 1$$

$$\Rightarrow \quad S = \frac{T\nu}{4}\int d^3\sigma\left(\dot{X}^\mu \dot{X}_\mu - \frac{2}{\nu^2}\{X^\mu, X^\nu\}\{X_\mu, X_\nu\}\right)$$

constraints: $\dot{X}^\mu \partial_a X_\mu = 0,$
$\dot{X}^\mu \dot{X}_\mu = -\frac{4}{\nu^2}\bar{h} = -\frac{2}{\nu^2}\{X^\mu, X^\nu\}\{X_\mu, X_\nu\}$

No quantization known for covariant membrane theory.

Light - front gauge

Define $\quad X^{\pm} = \frac{1}{\sqrt{2}}\left(X^0 \pm X^{10}\right)$

choose gauge $\quad X^+(\tau, \sigma_1, \sigma_2) = \tau$

constraints $\Rightarrow \quad \partial_a X^- = \dot{X}^i \partial_a X^i$

$$\dot{X}^- = \frac{1}{2}\dot{X}^i \dot{X}^i + \frac{1}{\nu^2}\{X^i, X^j\}\{X^i, X^j\}$$

$\rightarrow$ solve for X^-.

Hamiltonian $\rightarrow$

$$\boxed{\begin{array}{l} H = \frac{\nu T}{4}\int d^2\sigma\left(\dot{X}^i \dot{X}^i + \frac{2}{\nu^2}\{X^i, X^j\}\{X^i, X^j\}\right) \\ \text{constraint} \quad \{\dot{X}^i, X^i\} = 0 \end{array}}$$

Describes membrane in 11D, light-cone gauge.

To quantize, need

2.4 Membrane regularization

Idea: assume membrane $\approx \Sigma_2 \times \mathbb{R}$ (no topology change)

map functions on $\Sigma_2 \longrightarrow$ operators $\overset{N\times N}{(\text{matrices})}$

$$\{\cdot, \cdot\} \longrightarrow -\frac{iN}{2}[\cdot, \cdot]$$

Σ_2 finite $\longrightarrow$ matrices finite dimensional.

$$\frac{1}{4\pi}\int_{\Sigma_2} \cdot \longrightarrow \frac{1}{N}\mathrm{Tr}\ \cdot$$

$$\Rightarrow \quad H = \mathrm{Tr}\left(\frac{1}{2}\dot{X}^i - \frac{1}{4}[X^i, X^j][X^i, X^j] \right)$$

$X^{i=1,\dots,9}$ $N \times N$ matrices.

(set $2\pi T = 1$)

Constraint: $[\dot{X}^i, X^i] = 0$.

Explicit regularization depends on surface topology

<u>e.g. Sphere</u>: $\xi_1^2 + \xi_2^2 + \xi_3^2 = 1$

$$\{\xi_A, \xi_B\} = \varepsilon_{ABC}\,\xi_C$$

$\xi_A \rightarrow \frac{2}{N} J_A$. $N \times N$ generators of SU(2) irrep

(note, evals $+N/2 \rightarrow -N/2$, justifying $2/N$.)

$\{\cdot,\cdot\} \rightarrow [J_A, J_B] = i\,\varepsilon_{ABC} J_C$

symmetric, traceless

Spherical harmonics $Y_{\ell m}(\xi_1, \xi_2, \xi_3) = \sum_{A_i} t^{(\ell m)}_{A_1 \dots A_\ell}\, \xi_{A_1} \cdots \xi_{A_\ell}$

$$\rightarrow \mathbf{Y}_{\ell m} = \left(\frac{2}{N}\right)^{\ell} \sum t^{(\ell m)}_{A_1 \dots A_\ell} J_{A_1} \cdots J_{A_\ell}$$

$Y_{\ell m}$, $\ell < N \rightarrow \sum_{\ell=0}^{N-1} (2\ell + 1) = N^2$ DOF in $N \times N$ matrices.

– similar construction possible for other Σ_g's.

$\Rightarrow$ Final matrix model <u>independent</u> of topology.

– can generalize to supermembrane $\Rightarrow$ extra term $\frac{1}{2}\theta^T \gamma_i [X^i, \theta]$

2.5 Quantum Theory

string theory $\longrightarrow$ discrete spectrum $\left(M^2 = 0, \frac{1}{\alpha'}, \ldots\right)$

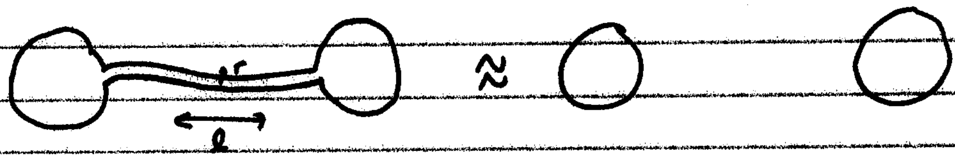
Matrix QM $\longrightarrow$ continuous spectrum

[note: removed in quantum bosonic thy, but reappears in susy thy]

[Apparent problem – stopped progress for 10 years]

Resolution: MT is 2nd quantized thy.

Thin tubes: ε energy



$$\Delta E \sim 2\pi r \ell \xrightarrow[r \to 0]{} 0$$

$\Rightarrow$ MT contains N-body states

membrane picture: matrices encode M2

BFSS : " " N D0's in IIA

$p^+ = N/R$,

as $N \to \infty$, get discrete light-cone quantized M-thy.

(DLCQ)

2.6 Objects in MT

Gravitons

Consider $N = 1$, $\ddot{X}^i = 0$, $X^i = y^i + v^i \tau$

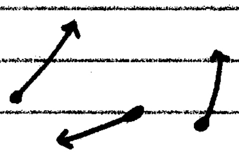
$\Rightarrow$ single graviton

fermions $\Rightarrow$ 128 polarization states = 128 components of supergraviton

Classical $N \times N$ solution

$$X^i = \begin{pmatrix} y^i_1 + v^i_1 \tau & & \\ & y^i_2 + v^i_2 \tau & \\ & & \ddots \end{pmatrix}$$

$\Rightarrow N$ gravitons



QM: $\exists!$ (up to polarization) bound state [Sethi/Stern 1998, Moore Nekrasov Shatashvili 2000] $\to$ prove Witten index $= 1$

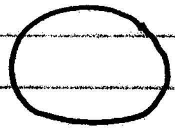
$p^+ = N/R$
KK state

wavefunction not known exactly

[some results on asymptotic form]

Membranes

invert matrix regularization $\Rightarrow$ macro. membranes from matrix DOF

e.g. spherical membrane  $X^i = \frac{2r}{N} J^i$

$$\left(\text{non-static: } \ddot{X}^i = -[[X^i, X^j], X^j] \right.$$
$$\left. \Rightarrow \ddot{r} \sim -r^3 \right)$$

Arbitrary membrane shape, topology
encoded in multipole moments $\mathrm{Tr}[X^a, X^b] X^{i_1} \cdots X^{i_k}$

$\longleftrightarrow$ D2 from D0-branes

2.7 Interactions

Simple example: 2 gravitons

Classical background: $\overset{\bullet}{y^i}$ $\quad \overset{\bullet}{z^i} \nearrow v^i$

$$B^i = \begin{pmatrix} y^i & 0^* \\ 0^* & z^i + v^i t \end{pmatrix}$$

Quantize off-diagonal $(*)$ DOF [1-loop calc.]
$\Rightarrow$ SHO's, $\omega(r, v)$

8 bosons: $\quad 6 \times \omega_b = r = \sqrt{(y-z)^2}$
[impose constraint] $\quad 1$ each $\times \omega_b = \sqrt{r^2 \pm 2v}$

16 fermions: $\quad 8$ each $\times \omega_f = \sqrt{r^2 \pm v}$

$$V_{eff} = \sum \omega_b - \frac{1}{2}\sum \omega_f = -\frac{15}{16}\frac{v^4}{r^7} + O\left(v^6/r^{11}\right)$$

agrees with SUGRA! [Calc by DKPS, evidence in BFSS paper]

More generally, background

$$B = \begin{pmatrix} \hat{X} & 0 \\ 0 & \tilde{X} \end{pmatrix} \quad \text{block-diagonal}$$

$\hat{X}$ $\tilde{X}$

$$\Rightarrow \quad V_{eff} = -\frac{15}{4r^7}\left(\hat{T}^{IJ}\tilde{T}_{IJ} - \frac{1}{9}\hat{T}^{I}{}_{I}\tilde{T}^{J}{}_{J}\right) - \frac{45}{r^7}\hat{J}^{IJK}\tilde{J}_{IJK} + \dots + \text{higher moments}$$

gives complete linearized SUGRA interaction

T^{IJ}, J^{IJK} $\quad I, .. \in \{+, -, 1, .., 9\}$

= stress tensor, M2 current

- defined in terms of $\hat{X}$, $\tilde{X}$

e.g. $J^{+ij} = -\frac{1}{6}\,\mathrm{Tr}\,[X^i, X^j] \rightarrow$ membrane current

($\rightarrow 0$ @ finite N, but higher moments interact thru SUGRA)

2.8 Status summary of Matrix Theory

- Well-defined Q. theory of SUGRA
- $\Rightarrow$ linearized SUGRA @ 1-loop
- $\Rightarrow$ Some nonlinear terms @ 2,3 loops (so far) [protected by NR theorems]
  [possible pert. problem @ 3-loops – R. Echols, Gray – may need nonpert. calc]
- Cpt. on T^d possible. $\Rightarrow$ $(d+1)$ SYM (T-duality)
- Believed to be complete nonperturbative Q Gravity theory
  – can use in principle to describe BH's, etc..

Open problems

- Find exact N – graviton bound state
- Derive complete nonlinear gravity theory (2-loops, nonpert?)
- Find covariant formulation
- Describe MT in general bg. space-time.

[WT, B Zwiebach 0311017]

3. String Field Theory

MT + AdS / CFT : nonperturbative string thy / QG
in fixed asymptotic ST bg.

Need 1 theory describing different BG's

SFT : nonperturbative target space formulation of ST.

Cleanest for open strings — recent progress following
Sen Conjectures

~~$\epsilon_\mu \alpha^\mu_{-1}|0;p\rangle$ is physical when $p^2 = M^2 = 0$~~

~~[for transverse polarizations — note $p\cdot\epsilon = 0$ from $Q \sim c_{-1}\, p\cdot\alpha$]~~

3.1 Witten's open Cubic string field theory

Witten proposed

$$S = -\frac{1}{2}\int \psi * Q\psi - \frac{g}{3}\int \psi * \psi * \psi$$

$\psi \in \mathcal{A}$ graded algebra

$* : \mathcal{A} \otimes \mathcal{A} \longrightarrow \mathcal{A}$ $\qquad G_{\psi * \phi} = G_\psi + G_\phi$

$Q : \mathcal{A} \to \mathcal{A}$ $\qquad G_{Q\phi} = 1 + G_\phi$

$\int : \mathcal{A} \to \mathbb{C}$ $\qquad \int \psi = 0, \quad G_\psi \neq 3.$

Desired properties:

a) $Q^2 = 0$

b) $\int Q\psi = 0. \quad \forall \psi \in \mathcal{A}$

c) $Q(\psi * \phi) = (Q\psi) * \phi + (-1)^{G_\psi}\, \psi * (Q\phi)$

d) $\int \psi * \phi = (-1)^{G_\psi \cdot G_\phi} \int \phi * \psi$

e) $(\phi * \psi) * \chi = \phi * (\psi * \chi)$

Given these properties, action invariant under

$$\delta\psi = Q\Lambda + g(\psi * \Lambda - \Lambda * \psi) \qquad G_\Lambda = 0$$

Motivation: this theory adds interactions to free theory,

$$S_{\text{Free}} = \int \psi * Q\psi \qquad \delta\psi = Q\Lambda$$

Action is off-shell with physical states ~ gauge equivalence classes of solutions $Q\psi$

Realization of SFT axioms

A: Take A = algebra of string fields

Formally: functionals $\psi[X(\sigma), b(\sigma), c(\sigma)]$

write using Fock space

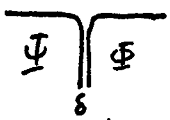
[like writing $f(x)$ as sum of SHO states]

$$|\Psi\rangle = \int d^{26}p \left[\phi(p)\,|0;p\rangle + A_\mu(p)\,\alpha_{-1}^{\mu}\,|0;p\rangle + \dots \right]$$

$$\left(\text{Ex. } |0\rangle \longrightarrow \psi_0 = C \exp\left(-\sum_n \frac{n}{2} x_n^2\right) \psi_0^{(g)} \right)$$

Q: usual BRST operator

$*$: gluing half-strings with δ-function overlap



$$(\psi * \Phi)[z(\sigma)] = \int \prod_{0 \le \sigma \le \frac{\pi}{2}} dx(\pi-\sigma)\, dy(\sigma)\ \delta(y(\sigma) - x(\pi-\sigma))\ \psi[x(\sigma)]\ \Phi[y(\sigma)]$$

$x(\sigma) = z(\sigma)$, $0 \le \sigma \le \frac{\pi}{2}$ $\qquad y(\sigma) = z(\sigma)$, $\frac{\pi}{2} \le \sigma \le \pi$

$\int$: Gluing $\Longrightarrow$ $\displaystyle\int \psi = \int \prod_{0\le\sigma\le\frac{\pi}{2}} dx(\sigma)\, dx(\pi-\sigma)\, \delta(x(\sigma)-x(\pi-\sigma))\, \psi[x(\sigma)]$

3.2 Oscillator Formulation

Formal integrals can be made precise using mode decomposition, oscillator representation.

Ghosts can be included (Grassman or bosonized form)

In Fock space,

$$\int \phi * \psi \longrightarrow \langle V_2|\left(|\phi\rangle \otimes |\psi\rangle\right) \qquad V_2 \in \mathcal{H}^* \otimes \mathcal{H}^*$$

$$\int \psi_1 * \psi_2 * \psi_3 \longrightarrow \langle V_3|\left(|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle\right) \qquad V_3 \in (\mathcal{H}^*)^3$$

~~give explicit forms of V_2, V_3 later.~~

Action becomes

$$S = -\frac{1}{2}\langle V_2|\left(|\psi\rangle \otimes Q|\psi\rangle\right) - \frac{g}{3}\langle V_3|\left(|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle\right).$$

~~Or written $-\frac{1}{2}\langle\psi|Q|\psi\rangle - \frac{g}{3}\langle\psi|\psi*\psi\rangle$, $\langle\psi|$ is BPZ dual of $|\psi\rangle$ $(z \to -1/z)$~~

$\Rightarrow$ Classical off-shell action for infinite family of space-time fields $\phi(p)$, $A_\mu(p)$, ...

3.2.1 Review:

δ functions, and the simple harmonic oscillator

Consider a simple harmonic oscillator with annihilation operator

$$a = -i\left(\sqrt{\frac{\alpha}{2}}\, x + \frac{1}{\sqrt{2\alpha}}\, \partial x\right)$$

and ground state

$$|0\rangle = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\frac{\alpha}{2}x^2}$$

Position basis states $|x\rangle$ have squeezed state form

$$|x\rangle = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left[-\frac{\alpha}{2}x^2 - i\sqrt{2\alpha}\, a^+ x + \frac{1}{2}(a^+)^2\right]|0\rangle$$

function $\rightarrow$ state correspondence:

$$f(x) \longrightarrow \int_{-\infty}^{\infty} f(x)\, |x\rangle \quad dx$$

In particular,

$$\delta(x) \longrightarrow \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(\frac{1}{2}(a^+)^2\right)|0\rangle$$

$$1 \longrightarrow \int dx\, |x\rangle = \left(\frac{4\pi}{\alpha}\right)^{1/4} \exp\left(-\frac{1}{2}(a^+)^2\right)|0\rangle.$$

Give squeezed state form for $\delta(x)$, 1.

These are <u>non-normalizable</u> states (almost) in $\mathcal{L}^2(\mathbb{R})$.

also: $\delta(x \pm y) \rightarrow \exp\left(\pm \frac{1}{2} a^+_{(x)} a^+_{(y)}\right)\left(|0\rangle_x \otimes |0\rangle_y\right)$

3.2.2 Two-string vertex $|V_2\rangle$.

Want to express $\langle V_2|(|\psi\rangle \otimes |\phi\rangle) = \int \psi * \phi$ in Fock space language

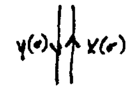
Recall

$$X(\sigma) = x_0 + \sqrt{2}\sum_{n=1}^{\infty} x_n \cos n\sigma$$

$$x_n = \frac{i}{\sqrt{2n}}(a_n - a_n^{+})$$

$$\psi[x(\sigma)] \to \psi[\{x_n\}].$$

$$\int \psi * \phi = \int \prod_{n=0}^{\infty} dx_n\, dy_n\, \delta(x_n - (-1)^n y_n)\, \psi[\{x_n\}]\, \phi[\{y_n\}]$$

encodes overlap condition $x(\sigma) = y(\pi - \sigma)$ 

$$\text{so } \langle V_2|_{matt} = (\langle 0| \otimes \langle 0|)\, \exp[-\underbrace{\sum_{n,m} a_n^{(1)*} C_{nm} a_m^{(2)*}}_{(\text{write } a^{(1)*}\cdot C \cdot a^{(2)*})}]$$

gives Fock space representation of $\langle V_2|_{matt}$.

Note: $|0\rangle$ is vacuum annihilated by a_0,

$$|0\rangle \sim C \exp[-x_0^2 - \sum_{n=1}^{\infty} \frac{n}{2} x_n^2]$$

For states like $|0;k\rangle$, use only $n,m \geq 1$.

$$(\langle 0| \otimes \langle 0|) \exp(-a_0^{(1)} a_0^{(2)}) \to \int d^{26}p\, (\langle 0;p| \otimes \langle 0;-p|)$$

extension to ghosts straightforward

$$
\begin{aligned}
X_1(\sigma) &= X_2(\pi-\sigma)\\
b_1(\sigma) &= b_2(\pi-\sigma)\\
c_1(\sigma) &= -c_2(\pi-\sigma)
\end{aligned}
$$

needed so total ghost # = 3.

$$\langle V_2| = \int d^{26}p\,\left(\langle 0_1; p| \otimes \langle 0_1; -p|\right)\,\left(c_0^{(1)} + c_0^{(2)}\right)$$

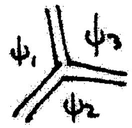
$$\exp\left[-\sum_{n=1}^{\infty}(-1)^n\left\{\alpha_n^{(1)}\frac{1}{n}\alpha_n^{(2)} + c_n^{(1)}b_n^{(2)} + c_n^{(2)}b_n^{(1)}\right\}\right]$$

same $|V_2\rangle$ can be derived from CFT approach

$$\int \psi * \phi \quad \longrightarrow$$

3.2.3 Three-string vertex $|V_3\rangle$

$$\langle V_3|\left(|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle\right) = \int \phi_1 * \phi_2 * \phi_3$$



can be computed in a very similar fashion.

Skipping details (see later)

$$\langle V_3| = \int d^{26}p^{(1)}\,d^{26}p^{(2)}\,d^{26}p^{(3)}\left(\langle 0_1; p^{(1)}| \otimes \langle 0_1; p^{(2)}| \otimes \langle 0_1; p^{(3)}|\right)$$

$$\delta\left(p^{(1)} + p^{(2)} + p^{(3)}\right)\; c_0^{(1)}c_0^{(2)}c_0^{(3)}$$

$$\bar{K}g\;\exp\left[-\frac{1}{2}\sum_{r,s}\left\{a_m^{(r)}V_{mn}^{rs}a_n^{(s)} + 2\,a_m^{(r)}V_{m0}^{rs}p^{(s)} + p^{(r)}V_{00}^{rs}p^{(s)}\right.\right.$$

$$\left.\left. + c_n^{(r)}X_{nm}^{rs}b_m^{(s)}\right\}\right]$$

$\bar{K} = 3^{9/2}/2^6$,

where V_{nm}^{rs}, X_{nm}^{rs} are calculable constants.

3.3 Calculating the SFT action

We can use $\langle V_2 |$, $\langle V_3 |$ to systematically compute terms in the SFT action

$$S = -\frac{1}{2}\langle V_2 | (|\psi\rangle \otimes |\psi\rangle) - \frac{g}{3}\langle V_3 | (|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle).$$

String field:

$$|\psi\rangle = \int d^{26}p \Big[\phi(p)\, |0_1; p\rangle + A_\mu(p)\, \alpha^\mu_{-1} |0; p\rangle$$
$$+ \chi(p)\, b_{-1} \cancel{c_0} |0_1; p\rangle + B_{\mu\nu}(p)\, \alpha^\mu_{-1}\alpha^\nu_{-1} |0_1; p\rangle + \dots$$

0 by FS gauge choice

Feynman–Siegel gauge choice: $b_0 |\Phi\rangle = 0$

Easy to show: good gauge near $|\psi\rangle = 0$.

(<u>Not</u> globally a good gauge).

Quadratic terms in S:

$$\langle V_2 | (|\psi\rangle \otimes |\psi\rangle) = \int d^{26}p \Big\{ \phi(-p)\left[\frac{p^2 - 1}{2}\right]\phi(p) + A_\mu(-p)\left[\frac{p^2}{2}\right] A^\mu(p) + \dots$$

Cubic terms:

$$\langle V_3 | (|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle)$$
$$= \int d^{26}p\, d^{26}q\, (\bar{K}g)\, e^{\left(\ln\frac{4}{3\sqrt{3}}\right)(p^2 + q^2 + p\cdot q)}\, \phi(p)\,\phi(q)\,\phi(-p-q)$$
$$+ \phi A_\mu A^\mu + \dots$$

$\left(V^{rs}_{00} = \delta^{rs} \ln \frac{4}{3\sqrt{3}}\right)$

generically, <u>nonlocal</u> cubic couplings connecting any 3 fields.

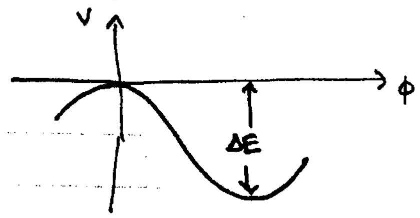
3.4 Sen conjectures

2 years ago, Sen suggested:

- bosonic open string describes D25-brane background.
- tachyon represents brane instability.
- SFT should ~~allow~~ give analytic description of true vacuum.

Precise conjectures:

1) classical SFT should have a locally stable nontrivial vacuum. Energy density should be $\Delta E = T_{25} = -1/2\pi^2 g^2$



2) Lower-dimensional branes should exist as Lorentz-breaking solitons in SFT.

3) Open strings should decouple in nontrivial vacuum (since D-brane is gone)

6. Evidence for Sen's conjectures: calculations in SFT

6.1 Level truncation and the stable vacuum.

SFT equation of motion: $Q\Phi + g\,\Phi * \Phi = 0.$

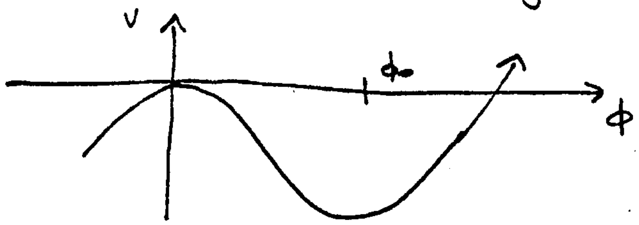
No nontrivial solution known analytically

Systematic approximation technique: Level truncation
- Drop all fields/(Int) above level L/(I) (10,7 at level ϕ)
- ~~Drop interactions of total level > I~~ ($2L \le I \le 3I$)

Simplest example: truncation at level $(L, I) = (0,0)$.

~~Drop all fields besides~~ just tachyon $\phi(p)$, set $p=0$
(Lorentz invariant solution)

$$V(\phi) = -\frac{1}{2}\phi^2 + g\bar{K}\phi^3$$

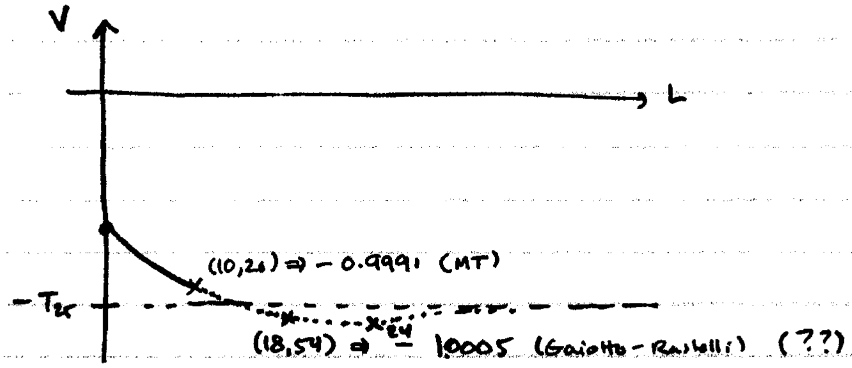


minimum at $\phi_0 = \dfrac{1}{3g\bar{K}}$,

$$V(\phi_0) = -\frac{1}{54}\frac{1}{g^2\bar{K}^2} = -\frac{2^{10}}{3^{10}}\frac{1}{g^2} \approx (0.68)\left(-\frac{1}{2\pi^2 g^2}\right)$$

Truncation to level 2, level 4 fields suggests convergence to stable vacuum [KS, 1990]
But - significance not understood at that time.

Higher level calcs:



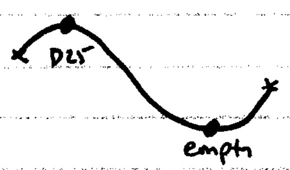
Use Padé, $\frac{1}{L}$ fit ⇒ min @ $L=24$, then → −1!

Summary:

- Numerical calcs have confirmed:
  - $E \to -T_{25}$ for stable sol'n
  - ∃ lump solutions w/ correct E
  - no open string states in tach vacuum.
- Rolling tachyon also studied (seems E → closed string gas)

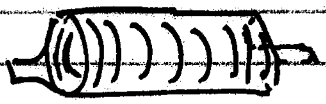
3.4 Outlook

Strong evidence OSFT describes multiple vacua



part of "open string landscape"

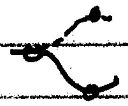
If OSFT a good Q. Thy, must include closed strings



$\Rightarrow$ OSFT may describe full string landscape, cosmology, etc.

Problems

- No analytic solutions [promising direction: VSFT (RSZ) $\Rightarrow$ D-branes = projectors ~ Noncomm. FT.]
- No precise mathematical formulation
  (e.g. what values of Φ are allowed, is there an $\mathbb{1}$?, ...)
  (norm cond, ...)
- Field redefinitions $\Rightarrow$ difficult to relate vacua
- No global gauge fixing (put x's)
  [some vacua may be "out of range"]
- = or > E vacua not yet understood
  - flat direction good to $O(\varepsilon)$ [Sen-Zw]
  - no 2× D25 sol'n yet



- Bosonic OSFT not well-defined QM thy
- Witten super OSFT has problems w/ picture changing
- Berkowitz theory promising
  - noncovariant
  - not yet tested @ loop level.