- Introduction to Mirror Symmetry.

* A physicist's view of Mirror Symmetry
  - Strings in backgrounds
  - Superstrings.
  - Calabi - Yau compactification.

- $S_P = \frac{1}{2\pi\alpha'}\int_\Sigma d^2\sigma\, \eta^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$, $X^{\mu=1,\cdots,d}: \Sigma \longrightarrow M^{1,d-1}$ Minkowski space.

Two ways to see this : ① String propagating world-sheet. ② Two d-scalar fields on Σ.

generalize

$$S_G = \frac{1}{2\pi\alpha'}\int_\Sigma d^2\sigma\, \eta^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} \quad , \quad X : \Sigma \longrightarrow M$$

NOT necessarily flat.

$G_{\mu\nu}$ → fluctuations in graviton.

Constraints : $T_{++} = T_{--} = 0$. after this constraints, it becomes string theory.

✓ Backgrounds ⟷ States in the spectrum.

scale-invariant backgrounds ⟷ graviton = massless states in the spectrum

(scale invariant $\sigma \to \lambda\sigma$. Mass perturbation theory $\eta_{\mu\nu} \sim G_{\mu\nu}$).

Massless States

$$L_0 = \frac{p^2}{2} + \sum_{n\geq 1} :\alpha_n \alpha_{-n}: - a = 0$$

$p^2 = 0$ (massless) $\Rightarrow \langle \psi | \sum :\alpha_n\alpha_{-n}: | \psi \rangle = 1$.

$\alpha_{-1}^i \bar{\alpha}_{-1}^j | p \rangle \longrightarrow$ graviton $\square\square \xrightarrow[\text{anti-symmetrizing}]{}$ anti-symmetric tensor $\xrightarrow[\text{taking trace}]{}$ scalar.

$B_{\mu\nu}(X) = -B_{\nu\mu}(X)$

$$S_B = \frac{1}{2\pi\alpha'}\int d^2\sigma\, \varepsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}(X)$$

B : a 2-form in space-time $\rightsquigarrow S_B = \frac{1}{2\pi\alpha'}\int_\Sigma X^*(B)$

Solve Perturbatively $\quad S_G = \frac{1}{2\pi\alpha'}\int \cdots G_{\mu\nu}(X) \xrightarrow[\text{large-radius limit}]{}$ free theory

$G \to \lambda G$, $\lambda \gg 1$
$\lambda G = (\eta + h)\lambda$

$\longrightarrow \alpha' \to 0$ (or zero-slope)

- What G, B are consistent ?
  G should satisfy Einstein's (gravity) equation.
  consistent backgrounds ⟷ conformally invariant actions.

$R_{\mu\nu} - \frac{1}{4} H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} = 0$, $\nabla^{\mu} H_{\mu\nu\rho} = 0$

$(d-26) - 3\alpha'(R - \frac{1}{12}H^2) = 0$. $H_{\mu\nu\rho} = [\partial_\mu B_{\nu\rho}]$, $H = dB$.

totally anti-symmetrizing

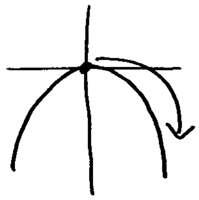
— We'll use $H=0$. ($\because \alpha' \to 0 \Rightarrow R_{\mu\nu} \to 0 \Rightarrow H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} = 0$)

- <u>Things we don't like in Bosonic string</u>
  - No fermions
  - Tachyon ($p^2 = 1$)
  - $d = 26$ ← Wasn't really a problem
  - complicated
  - No gauge bosons.

Solution to these = superstring

$\Box\varphi - \frac{m^2}{2}\varphi = 0$

$H = \int (\partial\varphi)^2 + \frac{m^2}{2}\varphi^2$



if $m^2 < 0$.

- <u>Superstrings</u>

$X^\mu(\sigma,\tau)$, $\psi^\mu(\sigma,\tau)$ ← fermionic (spinor) field.

$S_\psi = \frac{1}{2\pi\alpha'} \int d^2\sigma \, \frac{i}{2} \bar{\psi}^\mu \gamma \psi$ ← action for free-fermion.

- $T_{++} = T_{--} = 0$ is <u>NOT</u> enough to eliminate negative states.

eliminating longitudal excitation of string

since $\{\psi^\mu, \psi^\nu\} = i\eta^{\mu\nu}$

i.e. ψ^0 creates new Ghost !

One more constraint to solve this problem

$G_+ = \psi^\mu \partial_+ X^\mu = 0$

$G_- = \bar{\psi}^\mu \partial_- X^\mu = 0$

(Fermionic augumented version of $T_{++} = T_{--} = 0$.)

<u>Boundary conditions</u>

$\psi(\sigma + 2\pi, \tau) = \psi(\sigma,\tau)$, R .

$\psi(\sigma + 2\pi, \tau) = -\psi(\sigma,\tau)$, NS .

$\boxed{\text{Find}}$

- Critical $d = 10$.
- Lowest state in NS sector is a tachyon.
- In R sector, $\psi(\sigma,\tau) = \psi_0 + \sum_n b_n e^{in(\tau-\sigma)}$

$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$ <u>Clifford algebra</u>

Ground states are a representation of this $\Longrightarrow$ Spinors !

* Imposing a projection throws out half of states in the system.

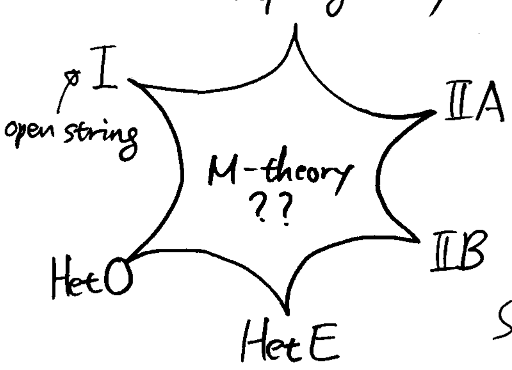
$\updownarrow$ $\boxed{\text{GSO projection}}$ consistently throws out half of states including tachyon

* Theory contains $\left.\begin{matrix} NS-R \\ R-NS \end{matrix}\right) \Rightarrow$ fermions.

Superstring		Heterotic string
• Fermions	✓	✓
• Tachyon	eliminated	✓
• $d = 10$	better	✓
• complicated		✓

• No gauge bosons ($\mathrm{II}A^{\circ}$, $\mathrm{II}B^{\times}$ have gauge bosons, but don't couple to anything)

1995 11-dim'l supergravity



$\boxed{d=10}$

$$L_0 = \frac{p^2}{2} + \text{oscillators} - a = 0 .$$

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{c}{12} n(n^2-1)\delta_{m+n,0}$$

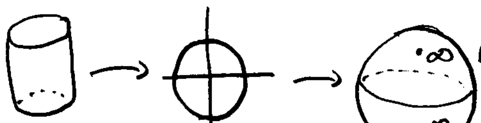
Need : $c = 26$ ← for bosonic string theory

15 ← for NGT (conformal invariant

State space is a representation space for

Virasoro $\oplus$ $\overline{\text{Virasoro}}$ (super-Virasoro algebra

To describe strings in 4-dim : $(X^{\mu=0,\cdots,3}, \psi^{\mu=0,\cdots,3}) \oplus$ (SCFT w/ $c = \bar{c} = 9$)



position   internal freedom   internal theory. Q good enough to make String thy

$$\boxed{\begin{aligned} &\sigma_\pm = \tau \pm \sigma \\ &z = e^{\tau + i\sigma} = e^{i(\sigma - i\tau)} \end{aligned}}$$

$$L_0 = \underbrace{L_0^{ST}}_{1+3} + \underbrace{\varphi \Box_K \varphi}_{6} + \cdots$$

space-time

A nice set of Q's

$$S = \frac{1}{2\pi\alpha'}\int_\Sigma d^2\sigma \Big(\partial_{\bar z}\varphi^I \partial_z \varphi^J g_{IJ}(\varphi) + \frac{i}{2} g_{IJ}(\varphi)\left[\psi_+^I \partial_- \psi_+^J + \psi_-^I \partial_+ \psi_-^J\right] + \frac{1}{4} R_{IJKL}\psi_+^I \psi_+^J \psi_-^K \psi_-^L \Big)$$

Conformal

$R_{IJ} + \cdots = 0$

$\boxed{\text{Supersymmetric Nonlinear } \sigma\text{-model}} \longrightarrow$

$\varphi : \Sigma \to K^6$

Q is an N=2 SCFT

- $R_{IJ} = 0$
- g_{IJ} : compatible w/ cpx structure on K
- g : Kähler

$\Longleftrightarrow$ Holonomy $\subseteq SU(3) \subseteq O(6)$

("=" ← Calabi-Yau)

Given a Calabi-Yau space K_6, find an $N=(2,2)$ SCFT Q.

$N=1$ spacetime SUSY $\Longleftrightarrow$ Q is $N=2$ SCFT w/ integer charge $q_L - q_R$

$N=2$ stability of perturbation problem.

Mirror Symmetry $K, \tilde{K}$ are mirror mfds, then $Q(K) \cong Q(\tilde{K})$

$Q \leftarrow K$, $Q \leftarrow \tilde{K}$: well-defined method

T-duality: Existence of different Lagrangians of string that lead to same CFT.

- To compactify "Internal" theory Q. ($N=2$ SCFT w/ $c=\bar{c}=9$ & integral $U(1)$ charges)

Conformal Field Theory

conformal transformations $z = f(z)$.

$$[L_n, L_m] = (n-m) L_{m+n} + \frac{c}{12} n(n^2-1)\, \delta_{m+n,0}$$

$\{L_0, L_1, L_{-1}\}$ generate $SL(2,\mathbb{R})$

- Find a basis: $L_0|\varphi\rangle = h|\varphi\rangle$, $\bar{L}_0|\varphi\rangle = \bar{h}|\varphi\rangle$ & $\exists! \; |0\rangle$ s.t. $L_0|0\rangle = 0 = \bar{L}_0|0\rangle$
- 1-1 correspondence: states $\longleftrightarrow$ operators (fields).

 field $\to \mathcal{O}(z,\bar{z}) \longleftrightarrow \mathcal{O}(0)|0\rangle$

- Fields form an algebra.

$$\mathcal{O}_1(z)\,\mathcal{O}_2(w) = \sum \frac{\mathcal{O}_3(w)}{(z-w)^{h_1+h_2-h_3}},$$ "Operator Product Expansion". converge in a punctured disc.

$$T(z) = \sum_n L_n z^{-n-2}, \quad L_0 = H+P$$

$$G^{\pm}(z) = \sum_n G^{\pm}_{n\pm a} z^{-(n\pm a)-\frac{3}{2}}, \quad G^{\pm}_0 = Q^{\pm}$$

$$J(z) = \sum_n J_n z^{-n-1}, \quad J_0 = R$$

$$[J_n, J_m] = \frac{c}{3} m\, \delta_{m+n,0}, \quad [L_n, L_m] = -mJ_{m+n}$$

$$[L_n, G^{\pm}_{m+a}] = \left(\frac{n}{2} - (m\pm a)\right) G^{\pm}_{m+n\pm a}$$

$$[J_n, G^{\pm}_{m+a}] = \pm G^{\pm}_{m+n\pm a}$$

$$\{G^{+}_{n+a}, G^{-}_{n-a}\} = 2L^{+}_{m+n} + (n-m+2a)J^{+}_{m+n} + \frac{c}{3}\left((n+a)^2 - \frac{1}{4}\right)\delta_{m+n,0}$$

$$a = \begin{cases} 0, & R \\ 1/2, & NS \end{cases}$$

$$L_n^{\dagger} = L_{-n}$$

$$J_n^{\dagger} = J_{-n}$$

$$(G^{\pm}_{n\pm a})^{\dagger} = G^{\mp}_{-n\mp a}$$

$\boxed{h, q : \text{eigenvalues of } L_0, J_0}$ $\quad G^{+}_{-\frac{1}{2}}|\phi\rangle = 0$ Chiral primary state

$$0 \le \langle\varphi| \{G^{-}_{1/2}, G^{+}_{-1/2}\} |\varphi\rangle = \langle\varphi| 2L_0 - J_0 |\varphi\rangle = 2h - q$$

$\Rightarrow h \ge q/2$. $\quad h = q/2 \Longleftrightarrow$ chiral primary

Also, $|q| \le c/3$ in any case.

$G^{-}_{-1/2}|\phi\rangle$ anti-chiral primary $\longrightarrow h = -q/2$.

- A, B: chiral primary

$$A(z)B(w) = \sum_C \frac{C(w)}{(z-w)^{h_A+h_B-h_C}}$$ & $\nexists$ singular product on chiral primary

$\Rightarrow h_C \ge h_A + h_B \Rightarrow q_C = q_A + q_B$ if A, B are both chiral primary.

$(A\cdot B)(w) = \lim_{z\to w} A(z)B(w)$ ← defines $A\cdot B$ as a chiral primary.

⟶ Ring structure on chiral primary states.

✓ Chiral Primary w/ $|q|=1$, $h=1/2$.

$A:(c,c)$ w/ $q=\bar{q}=1 \Rightarrow \mathcal{O}=G^-_{-1/2}\bar{G}^-_{-1/2}A : h=\bar{h}=1$ (marginal)

$S \to S + t\int d^2z\,\mathcal{O}$ (exactly) truly marginal.

Example 0. $S=\int d^2z \left(\bar\partial\varphi\partial\bar\varphi + i(\bar\psi^-\partial\psi^- + \bar\psi^+\bar\partial\psi^+)\right)$

$T(z) = \partial\varphi\partial\bar\varphi + \frac{1}{2}(\bar\psi^+\partial\psi^+ + \psi^+\partial\bar\psi^+)$, $c=3$

$J(z) = \frac{1}{4}\bar\psi^+\psi^+$, $G^+(z)=\frac{1}{2}\bar\psi^+\partial\varphi$, $G^-(z)=\frac{1}{2}\psi^+\partial\bar\varphi$

ψ^+ charges $(1,0)$, weights $(1/2,0)$, (c,c): $(1,\psi^+,\psi^-,\psi^+\psi^-)$ spectrum of weights...

free action. free cpx bosons/fermions ⟹ SCFT exists.

Example 1. $2\pi\alpha' S=\int \Big\{\frac{1}{2}(g_{IJ}+iB_{IJ})\partial\varphi^I\bar\partial\varphi^J + g_{i\bar{j}}(\bar\psi^{\bar{j}}_- D_z\psi^i_- + \psi^{\bar{j}}_+ D_{\bar{z}}\psi^i_+) + R_{i\bar{k}j\bar{l}}\,\psi^i_+\psi^j_-\bar\psi^{\bar{k}}_-\bar\psi^{\bar{l}}_+\Big\}$

- $c=3n$.
- $(c,c) \sim \bigoplus_{p,q} H^p(\Lambda^q TM)$
- $(a,c) \sim \bigoplus_{p,q} H^p(\Lambda^q TM)$

(starting from g, we can inductively correct g and in the limit this action is CFT)

$\boxed{\text{Fact}}$ In extremely low energy, every theory becomes either CFT or trivial theory

Example 2. [Landau-Ginzburg theory.]

$W(\phi) = \frac{1}{k+2}\phi^{k+2}$

$L_W = \frac{\partial^2 W}{\partial\phi^2}\psi_+\psi_- + \left|\frac{\partial W}{\partial\phi}\right|^2 + h.c.$

(supersymmetric, but NOT conformal. W is NOT corrected under renormalization. Only kinetic terms suffers from RG.)

$(\varphi,\psi_+,\psi_-) \longrightarrow e^{-i\alpha\frac{1}{k+2}}(\varphi,\psi_+,\psi_-)$ corrects R-symmetry.

- $c=\frac{3k}{k+2} < 3 \Rightarrow$ Central charge is less than that of free theory.
- $(c,c) \sim \frac{\mathbb{C}[\varphi]}{\partial W(\varphi)}$ $(1,\varphi,\cdots,\varphi^k)$, $R(\varphi)=\frac{1}{k+2}$
- Representations of Super-Conformal algebra with $c<3$.
  - $C_k=\frac{3k}{k+2}$ ← only solutions (minimal models)
  - Can exactly compute spectrum, correlator.

Thm [Yau] M : compact $6_{\mathbb{R}}$-mfd. $c_1(M) = 0$ Ricci-flat condition

Given complex structure J on M, and a class $k \in H^{1,1}(M)$ appropriate, then $\exists !$ Kähler metric on (M, J) such that

$$J_{i\bar{j}} = -i g_{i\bar{j}} dz^i \wedge dz^{\bar{j}}, \quad [J] = k \leftarrow \text{Kähler class.}$$

Rmk. Given k, we can compute higher-order corrections in Einstein-metric.

$$\boxed{\text{given}}\ (M, J, k) \xrightarrow{\text{gives}} \text{CFT} \to Q \qquad \text{vary } k \to \text{family of } Q$$

parametrized by k & J.

$$\boxed{\begin{array}{l}\text{prediction for}\\ \text{① central charge}\\ \text{② chiral ring}\end{array}}$$

$$(c,c) \sim \bigoplus_{p,q} H^p(\Lambda^q TM) \xleftrightarrow{\text{marginal}} H^1(TM) \leftarrow \text{deformation of complex structure}$$

NOT obstructed in Calabi-Yau.

$$(a,c) \sim \bigoplus_{p,q} H^p(\Lambda^q TM) \xleftrightarrow{\text{determinants}} H^1(T^*M) = H^{1,1}(M) \leftarrow \text{" " Kähler "}$$

$$S_B = \int_\Sigma \varphi^*(B)$$

(a) invariant under $B \to B + d\Lambda$

$[k] = [B + iJ] \in H^{1,1}(M, \mathbb{C})$.

(b) $B \to B + \eta$, $\eta \in H^2(M, \mathbb{Z})$

$S \to S + \eta$, $B \in H^2(M) / H^2(M, \mathbb{Z})$

M_k, $C_k = \frac{3k}{k+2}$, discrete $\mathbb{Z}_{k+2}$-symmetry.

correspond

$W = \frac{1}{k+2}\phi^{k+2}$, $R(\phi) = \frac{1}{k+2}$

$\bigotimes_i M_{k_i}$, SCFT, $c = \sum_i \frac{3k_i}{k_i+2} \stackrel{\text{by choosing } k_i}{=} 9 \leftarrow$ Good! Still need integral $U(1)$-charges !!

Gefner (?) $Q := \left(\bigotimes_i M_{k_i} / G\right)$ has integral $U(1)$ charges $\left(G \subset \prod_i \mathbb{Z}_{k_i+2}\right)$

<u>exactly solve string compactification!</u>

- 5 copies of $C_3 = \frac{9}{5} \to c = 9$ theory, $Q = (3^5)/\mathbb{Z}_5$

$L =$ kinetic terms $+ \cdots$, $W = \frac{1}{5}\sum_i \phi_i^5 \leftarrow$ looks like discrete symmetries of C

$(1, \phi, \cdots, \phi^k)$ chiral generators, $\phi^{k+1} = 0 \to \phi_i^4 = 0$.

(c,c) : $\left(1, \prod_i \phi_i^{p_i}, \prod_i \phi_i^{q_i}, \prod_i \phi_i^3\right)$

$\sum p_i = 5 \quad \sum q_i = 10$

$R = 0, \quad 1, \quad 2, \quad 3$

$\mathbb{C}P^4$, $f(z)=0$. $z_1, \cdots, z_5$: homogeneous coordinates.

$c_1 = 0 \Leftrightarrow f$: homogeneous polynomial of degree 5.

$\exists!$ complex structure on $\{f=0\}$ induced from complex structure of $\mathbb{C}P^4$ and defined by coefficients of f.

$\rightsquigarrow$ 101-dim'l space of complex structure parametrized by homogeneous variables z_i. Which size of Kähler class reduce it to $(3^5)/\mathbb{Z}_5 = Q$?

$\boxed{f(z) = \sum_i z_i^5}$ $\quad h^{1,1} = 1$

- M_k : $\mathbb{Z}_{k+2}$ - discrete symmetry

Fact Any CFT with discrete symmetry, we can gauge it.

$M_k/\mathbb{Z}_{k+2} \overset{\substack{L \to L \\ R \to -R}}{\simeq} M_k$. $\forall Q/\Gamma$ Γ discrete symmetry group.

Strings can propagate in orbifold

$$\left(\prod M_k/G\right)/H \overset{\substack{L \to L \\ R \to -R}}{\simeq} \left(\prod M_k/G\right)$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ M/H & \overset{\text{Mirror !!}}{\longleftrightarrow} & M \end{array}$$

- $\mathbb{C}P^4$. $M = \{f(z)=0\} \subset \mathbb{C}P^4$, $Q(M/\mathbb{Z}_5^3) \overset{\substack{L \to L \\ R \to -R \\ c \to a \\ a \to c}}{\simeq} Q(M)$

$x \to -x$

cpx def $\longleftrightarrow$ Kähler def.

✓ Any CY that can be formed as a hypersurface in weighted $\mathbb{C}P^4$ has a mirror.

✓ Any complete intersection in toric variety M

$\exists W$ s.t. • $h^{p,q}(W) = h^{d-p,q}(M)$

• ring structure of chiral rings agree.

Conj $\forall$ Calabi-Yau M, $\exists$ mirror W.

rigid complex structure $\rightsquigarrow$ No Kähler deformation $\rightsquigarrow$ No Kähler form.

- SYZ ('97) Propose construction.

let M be a CY such that $\exists$ mirror W.

$$\Rightarrow \quad \begin{array}{ccc} M & \longleftarrow & T^3 \\ \downarrow & & \text{calibrated} \\ B & & \end{array} \quad \& \quad \begin{array}{ccc} W & \longleftarrow & T^3. \\ \downarrow & & \uparrow \\ B & & \text{dual torus} \end{array}$$

What is fibration in physics ? Don't know !

$$2\pi\alpha' \mathcal{L} = \frac{1}{2}\int (g_{IJ} + iB_{IJ})\partial\varphi^I \bar{\partial}\varphi^J + i g_{i\bar{j}}(\bar{\psi}_-^{\bar{j}} D\psi_-^i + \psi_+^{\bar{j}} \bar{D}\psi_+^i) + R_{i\bar{j}k\bar{l}}\psi_+^i \psi_-^j \bar{\psi}_+^k \bar{\psi}_-^{\bar{l}}$$

$\forall \mathcal{L}$, $\exists$ metric w/ Kähler such that $\mathcal{L}$ becomes SCFT.

- Topological Twisted Models.

Symmetry groups $U(1)_R \times U(1)_L \times U(1)_E$ (Euclidean ($SO(2)$))

$g_R \quad g_L \quad g_E$.

$\boxed{T'(z) \text{ have } c=0}$

Define $g_{E'}^2 = g_E^2 g_R g_L$, $T'(z) = T(z) \pm \frac{1}{2}\partial J(z)$ energy-momentum tensor

$\begin{matrix}+\\-\end{matrix} \times \begin{matrix}R\\L\end{matrix} \rightarrow 4$ twists, complex conjugate pair them $\rightarrow$ 2 twists left.

Q_+	$\bar{Q}_+$
$h=0$	$h=1$
spin: 0	1

Reason of doing this: $Q_+^2 = 0$ (spin 0) easily extends to any curved world-sheet $\longleftrightarrow$ like BRST charge $\updownarrow$ $d^2=0$ in $\mathcal{H}$.

Topological: find $Q_+|\psi\rangle = 0 \rightarrow$ decide $|\psi\rangle \sim |\psi\rangle + Q_+|\chi\rangle$

- Equivalence classes $\longleftrightarrow$ chiral primaries $\leftarrow$ This theory isolates chiral primary states and enable us to deal with finite dim'l Hilbert space.
- $\{Q^+, G^-(z)\} = 2T'(z)$ $\boxed{\text{world-sheet metric}}$

$\Rightarrow$ correlators are independent of $h_{\alpha\beta}$

For any $O(z)$, $\partial O(z)$ is Q-exact.

$\Rightarrow$ correlators are independent of position!

$\therefore$ finite set of operators with $h_{\alpha\beta}$, position independent correlation functions. (operators $A \xrightarrow{\text{suitable choice}}$ original operators)

- $\mathcal{L}' = \mathcal{L} + \{Q_+, \Lambda\}$ yields the same TFT.

$\int D\varphi D\psi \cdot e^{iS}$, $S[\varphi] = S[\varphi + \delta\varphi]$ allows symmetry $\xrightarrow{\text{lead to}}$ "Ward identities" of correlation ftn $\int D\varphi D\psi e^{iS}$

(if) Grassman parameter $\Rightarrow$ +super

If $\nexists$ fixed points of $\Rightarrow \int d\psi_0$ (independent of ψ_0) $= 0$ by rules of Grassmannian

orbits

Hence, $\int D\varphi D\psi \cdot e^{iS} \longrightarrow \int_{[\varphi] \text{ s.t. } \delta\varphi = 0} D\varphi D\psi\, e^{iS}$ $\leftarrow \nexists$ measure factor to consider.

$\boxed{\text{Fact}}$ normal bundle = super-bundle.

Witten

A-model = TFT obtained by

B-model = " " "

	ψ_+ (right)	ψ_- (left)	
A-model	+	−	twisting
B-model	+	+	"

A-model

$\chi \in \Gamma(\pi \varphi^* TM)$

$\psi_+, \overline{\psi_-}$ become scalars which is a section $\Gamma(\pi \varphi^*(T^{(1,0)}M)), (T^{(0,1)}M)$

$\delta_\eta \varphi^i = -\sqrt{2}\eta_- \chi^i$, $\delta \bar{\varphi}^{\bar{i}} = \sqrt{2}\,\bar{\eta}_+ \chi^{\bar{i}}$, $\quad \overset{Q_A}{\eta_- = \bar{\eta}_+ = \eta}$.

$\delta \chi = 0$. $\delta \bar{\psi}_z^{\bar{i}} = -i\sqrt{2}\,\partial \varphi^{\bar{i}} \eta_- + \sqrt{2}\bar{\eta} + \chi^{\bar{j}} \Gamma^{\bar{i}}_{\bar{j}\bar{k}} \bar{\psi}_z^{\bar{k}}$

$\mathcal{L} = i\{Q_A, V\} - i\varphi^*(K)$ ⟵ other than choice of Kähler classes

$\varphi^*(K) = (B_{i\bar{j}} + i g_{i\bar{j}})(\partial\varphi^i \bar{\partial}\varphi^{\bar{j}} - \bar{\partial}\varphi^i \partial \varphi^{\bar{j}})$, choice of metric is completely irrelevant, i.e. independent of complex structures.

$V = g_{\bar{i}j}(\bar{\psi}_z^{\bar{i}} \bar{\partial}\varphi^j + \partial \varphi^{\bar{i}} \psi_{\bar{z}}^j)$

$\Rightarrow$ A-model is <u>independent</u> of complex structure

Dependence on Kähler class, $K = \sum_a 2\pi t_a e_a$.

Pick basis for H_2, E_a. In sector where image is $\sum n^a E_a$,

$e^{iS} \sim e^{2\pi i \sum t_a n^a} = \prod q_a^{n_a}$, $\quad \boxed{t \to \infty \Leftrightarrow \text{large radius limit} \rightarrow \bar{\partial}\varphi^i = 0 \text{ critical case.}}$

Path integral $\xrightarrow{\text{reduces}} \sum_{n_a} q_a^{n_a} \int_{\mathcal{M}(n)}$ ⟵ moduli space of holomorphic maps $\mathbb{P}^1 \to M$ with multi-degree n.

- $\mathcal{O} = \sum f_{i_1,\cdots,i_k,\bar{j}_1,\cdots,\bar{j}_l}(\varphi)\, \chi^{i_1} \cdots \chi^{i_k} \chi^{\bar{j}_1} \cdots \chi^{\bar{j}_l}$ $\quad \boxed{\text{virtual dimension of } \mathcal{M}(n) = d + n\cdot c_1(M) = d \text{ for CY.}}$

$\{Q_A, \mathcal{O}\} = -\mathcal{O}_{d\eta}$, $\eta = \sum f_{\cdots}\, d\varphi^{i_1} \wedge \cdots \wedge d\varphi^{\bar{j}_l}$

↳ 1-1 correspondence w/ DeRham cohomology.

$\langle \prod_i \mathcal{O}_i \rangle = 0$ unless $\sum k_i = \sum l_i = -\frac{d}{2}\chi(\Sigma)$.

$\eta_i \in H^{1,1}(M)$ $\quad \langle \prod_{i=1}^{d} \mathcal{O}_{\eta_i}(z_i) \rangle = \sum_{n_a} q_a^{n_a} \int_{\mathcal{M}(n)} \prod \mathcal{O}_{\eta_i}$ ⟵ $\underline{\text{Clemens}}$ is $< \infty$

$\eta_i \xleftrightarrow{\text{relate}} H_i \in H_{d-2}(M)$

# of hol. maps : $\mathbb{P}^1 \to M$ s.t. $\varphi(z_i) \in H_i$

B-model

$$\begin{cases} \chi^{\bar{i}} = \bar{\psi}_+^{\bar{i}} + \bar{\psi}_-^{\bar{i}} \\ \theta = g_{i\bar{i}}(\bar{\psi}_+^{\bar{i}} - \bar{\psi}_-^{\bar{i}}) \end{cases}$$

B-model correlators are independent on Kähler structure and dependent on complex structure.

$\delta \varphi^i = 0$

$\delta \varphi^{\bar{i}} = \sqrt{2}\eta \chi^{\bar{i}}$

$\delta \chi^{\bar{i}} = 0$

$\delta \theta_i = 0$

$\delta \rho^i = i\sqrt{2}(d\varphi)\eta$

fixed points set $= \{d\varphi = 0\}$ = constant maps

$\rightarrow$ inconsistent as QFT unless M is CY.

$V \in \bigoplus H^k(\Lambda^\ell TM)$

$\langle \prod \mathcal{O}_{V_a}(\cdot) \rangle = \int_M V_1 \wedge \cdots \wedge V_a \cdot \underbrace{\Omega \wedge \bar{\Omega}}_{\text{Complex Structure}}$ $\qquad T' = T \overset{A}{\underset{B}{\pm}} \frac{1}{2} \partial J$

M, W : mirrors $\overset{\text{Mirror Symmetry}}{\Longrightarrow}$ $\underset{?}{\Longleftarrow}$ $A(M) \simeq B(W)$, $\quad q \longleftrightarrow z$.

- **Linear Sigma-model**

$N=2$ SUSY abelian gauge theory (in 2 dim.)

$\phi = \varphi + \theta^+ \psi_+ + \theta^- \psi_- + \theta^+ \theta^- F$ $\quad$ chiral super-field.

$\int d^4\theta\, \bar{\phi} \phi = \partial \varphi \bar{\partial} \varphi + \bar{\psi}_+ \partial \psi_+ + \bar{\psi}_- \partial \psi_- + |F|^2$.

$V = \theta^- \bar{\theta}^- V_z + \theta^+ \bar{\theta}^+ V_{\bar{z}} - \theta^- \bar{\theta}^+ \sigma - \theta^+ \bar{\theta}^- \bar{\sigma} + i \theta^2 \bar{\theta}^+ \lambda_+ + \cdots + \theta^4 D$.

$\Sigma = \bar{D}_+ D_- V = \sigma + \theta^+ \bar{\lambda}_+ + \bar{\theta}^- \lambda_- + \theta^+ \bar{\theta}^- (D + if)$, $\quad f = \partial_0 V_1 - \partial_1 V_0$

$V_{a=1,\cdots,(n-d)}$, $\quad \Phi_{i=1,\cdots,n}$, $\quad Q_i^a$, $\quad \phi_i \to e^{i Q_i^a \alpha_a} \phi_i$

$\boxed{\int d^2\theta\, W(\phi)}$ $\quad \int d\theta^+ d\bar{\theta}^- \tilde{W}(\Sigma)$, $\quad \tilde{W}(\Sigma) = \sum_a \Sigma_a t_a$, $\quad t_a = i r_a + \frac{\theta_a}{2\pi}$.

$U = \sum_i |F_i|^2 + \sum_a D_a^2 + \sum_{a,b} \bar{\sigma}_a \sigma_b \sum_i Q_i^a Q_i^b |\varphi_i|^2$

$F_i = \frac{\partial W}{\partial \varphi_i}$, $\quad D_a = \sum \underbrace{Q_i^a |\varphi_i|^2}_{} - r_a$

↳ Hamiltonian Moment map !

Gauge-inequivalent vacua $\longleftarrow$ GIT-quotient.

$\{ D_a = 0 \} / U(1)^{n-d} = \mathbb{C}^n /\!/ U(1)^{n-d} = \mathbb{C}^n \setminus F^{(r)} / (\mathbb{C}^*)^{n-d}$.

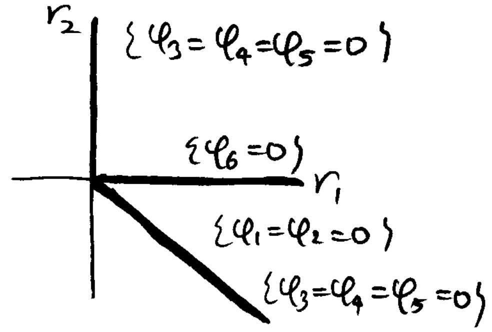
- $\phi_i \longrightarrow \prod_a \lambda_a^{Q_i^a} \phi_i$, $\quad Q_i^a$: combinatorial data (toric variety).

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6		φ_0
Q_1	0	0	1	1	1	1	r_1	-4
Q_2	-1	-1	0	0	0	2	r_2	0

add line bundle $\downarrow$ toric variety

$\boxed{\text{noncompact CY}}$

$\varphi_0 = 0 \longrightarrow$ original toric variety



$W = \varphi_0 f(\varphi)$

$F_0 = \partial W / \partial \varphi_0 = f(\varphi) = 0$

$F_i = \varphi_0 \frac{\partial f}{\partial \varphi_i} \Rightarrow \varphi_0 = 0$

$$\Delta A = \sum_a (\underbrace{\sum_i Q_i^a}_{=0}) n_a , \quad n_a = \frac{1}{2\pi}\int_\Sigma f_a .$$

$Q_R - Q_L$

$$q_a = e^{2\pi i t_a} = e^{i\theta_a - 2\pi r_a} , \qquad \beta(r_a) \propto \sum_i Q_i^a$$

$r_1 \ll 0$

$$D_1 = 0 \Rightarrow \varphi_0 \neq 0 \xRightarrow{F_i = 0} \varphi_i = 0 \qquad D_1^2 = \left(\sum |\varphi_i|^2 - 4|\varphi_0|^2 - r_1 \right)^2$$

$$\boxed{W = f(\varphi)} / \mathbb{Z}_4 , \qquad f(\varphi) = \underbrace{(\varphi_1^8 + \varphi_2^8)\varphi_6^4}_{k=6} + \underbrace{\varphi_3^4 + \varphi_4^4 + \varphi_5^4}_{k=2}$$

$\sum \frac{3k}{k+2} = 9.$ minimal model $\overset{\text{correspond}}{\sim}$ some deformation of CY.

$$\mathcal{M}_n = \varphi_i : \mathbb{P}^1 \longrightarrow \mathbb{C}^n \quad \text{non-cpt.}$$

$\mathcal{M}$ compactify

$$\{ D^{-1}(0) \subseteq \mathbb{C}^{\sum(d_i+1)} \} / U(1)^{n-d}$$

φ_i : degree $\sum n_a Q_i^a = d$.

$f(\varphi) \equiv 0.$ $\qquad f = \sum z_i^5 - 5\psi \prod z_i$

$$\boxed{1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -5} \qquad \langle 6^3 \rangle = \frac{5}{1+5^5 q} , \qquad \frac{1}{1-(5\psi)^5}$$

$$W = \varphi_0 \sum \varphi_i^5 .$$